

U. S. Department of Commerce

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Coast and Geodetic Survey

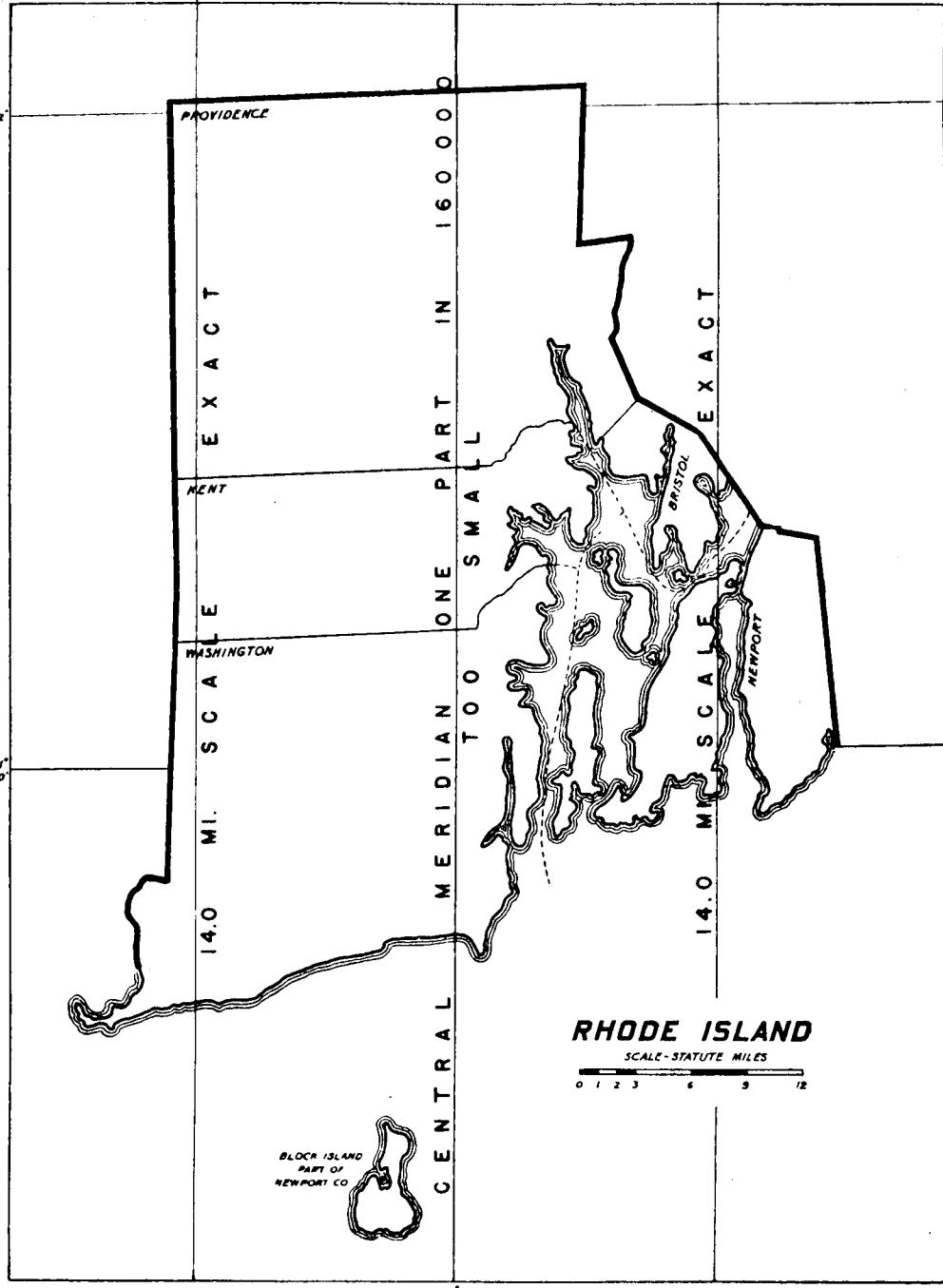
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**PLANE COORDINATE PROJECTION TABLES
RHODE ISLAND**



71°30'



Foreword

The plane coordinate system used in this State is based on the transverse Mercator projection using a reduced scale for the central meridian of the zone. The tables in this publication are to be used for the conversion of geographic positions to plane coordinates or plane coordinates to geographic positions. The constants of the projection are listed with the tables.

The methods of computation have been designed for machine calculation. All of the functions that are required are given in this publication.

The formulas and sample computations which follow show the general methods for computing either type of coordinates.

Plane coordinates from geographic positions

$$x = x' + 500,000$$

$$x' = H \cdot \Delta\lambda'' \pm a$$

$$y = y_0 + v \left(\frac{\Delta\lambda''}{100} \right)^2 \pm c$$

Grid azimuth = geodetic azimuth - $\Delta\alpha$ - second term

$$\Delta\alpha'' = \Delta\lambda'' \sin \phi + g$$

where

y_0 , H , v , and a are based on the latitude
of the geographic position,

and

b , c , and g are based on $\Delta\lambda''$.

$$\Delta\lambda'' = \text{Central Meridian} - \lambda$$

and

$\Delta\alpha''$ is the convergence of the meridian at the station with respect to the Central Meridian.

The second term for the reduction of geodetic to grid azimuths may be neglected for most work. However, for lines five miles or more in length if the same degree of accuracy is desired as is obtained by geographic computations, this term should be evaluated and used.

$$\text{Second term} = \frac{(y_2 - y_1)(2x'_1 + x'_2)}{(6 \rho_o^2 \sin l'') g}$$

Geographic positions from plane coordinates

$$P(x'/10,000)^2 + d = V(\Delta\lambda''/100)^2 + c$$

$$y_o = y - P(x'/10,000)^2 - d$$

Obtain the latitude from the table of y_o .

Use latitude to obtain H from the table.

$$x' = x - 500,000$$

$$\text{approximate } \Delta\lambda'' = x' \div H.$$

Determine a from latitude and b from approximate $\Delta\lambda$
then

$$\Delta\lambda'' = (x' + a b) \div H$$

$$\Delta\alpha'' = Mx' - e$$

M is based on the y, and e on the x and y of the plane coordinates.

PLANE COORDINATES ON TRANSVERSE MERCATOR PROJECTION
(Condensed form for calculating-machine computation)

State Rhode Is./and Zone — Central meridian $71^{\circ} 30' 00.''000$

Station	Draper, 1932	Knew, 1943	
ϕ	$41^{\circ} 32' 24.848$	$41^{\circ} 23' 53.266$	
λ	$71^{\circ} 16' 00.833$	$71^{\circ} 37' 13.730$	
$\Delta\lambda - \text{Central mer.} - \lambda$	$+ 0^{\circ} 13' 59.167$	$- 0^{\circ} 07' 13.730$	
$\Delta\lambda''$	$+ 839.''167$	$- 433.''730$	
$V \left(\frac{\Delta\lambda''}{100} \right)^2$	70.420	18.812	
H	76.048	680	76.214
V	1.222	636	1.221
a	b	-0.862	$+0.541$
$x' = H \cdot \Delta\lambda \pm ab$	$+ 6.3$	817.08	$- 33.056.45$
$V \left(\frac{\Delta\lambda''}{100} \right)^2 \pm c$		86.09	22.98
Tabular y	166	477.51	114
x	563	817.08	698.09
y	166	563.60	943.55
$\Delta\alpha''$	$+ 556.49$	$- 114$	721.07
$\Delta\alpha$	$+ 0^{\circ} 09' 16.5$	$- 0^{\circ} 04'$	46.8
Geod. Az. to Az. Mk.	41	37	44
Grid Az. to Az. Mk.	41	27	54
	341	48	58

$$x = x' + 500,000$$

$$y = \text{Tab. } y + V \left(\frac{\Delta\lambda''}{100} \right)^2 \pm$$

$\Delta\alpha'' = \Delta\lambda'' \sin\phi + g$
Grid Az. = Geod. Az. $- \Delta\alpha$

H and $V = \text{Tab. } H$ and Tab. V .

When ab is $-$, decrease $H \Delta\lambda$ numerically.
 g increases $\Delta\lambda'' \cdot \sin\phi$ numerically.

GEODETIC POSITIONS FROM TRANSVERSE MERCATOR COORDINATES
(CALCULATING MACHINE COMPUTATION)

STATE - ZONE Rhode Island

Station Draper, 1932

X	563,817.08	Y	166,563.60
C	- 500,000.00	$P\left(\frac{X'}{10,000}\right)^2 + d$	- 86.08
X'	+ 63,817.08	Y_0	166,477.52
P	2.11375	Approx. $\Delta\lambda = X' \div H$	+ 839 "
d	0.00	$\Delta\lambda = (X' + ab) \div H$	+ 839.167
H	76.048 680	$\Delta\lambda$	+ 0° 13' 59.167
a b	- 0.862	Central Meridian	71 30 00.000
φ	41° 32' 24"848	$\lambda = C.M. - \Delta\lambda$	71° 16' 00.833

Station Knew, 1943

X	466,943.55	Y	114,721.07
C	- 500,000.00	$P\left(\frac{X'}{10,000}\right)^2 + d$	- 22.98
X'	- 33,056.45	Y_0	114,698.09
P	2.10320	Approx. $\Delta\lambda = X' \div H$	- 434 "
d	0.00	$\Delta\lambda = (X' + ab) \div H$	- 433.730
H	76.214 944	$\Delta\lambda$	- 0° 07' 13.730
a b	- 0.898	Central Meridian	71 30 00.000
φ	41° 23' 53.266	$\lambda = C.M. - \Delta\lambda$	71° 37' 13.730

Station

X		Y	
C	-	$P\left(\frac{X'}{10,000}\right)^2 + d$	-
X'		Y_0	
P		Approx. $\Delta\lambda = X' \div H$	"
d		$\Delta\lambda = (X' + ab) \div H$	"
H		$\Delta\lambda$	° ′ ″
a b		Central Meridian	° ′ ″
φ	° ′ ″	$\lambda = C.M. - \Delta\lambda$	° ′ ″

Station

X		Y	
C	-	$P\left(\frac{X'}{10,000}\right)^2 + d$	-
X'		Y_0	
P		Approx. $\Delta\lambda = X' \div H$	"
d		$\Delta\lambda = (X' + ab) \div H$	"
H		$\Delta\lambda$	° ′ ″
a b		Central Meridian	° ′ ″
φ	° ′ ″	$\lambda = C.M. - \Delta\lambda$	° ′ ″

When a, b is $+$, decrease X' numerically
 $-$, increase X' numerically

Constants for Rhode Island

Central Meridian	71° 30' 00" 000
log R	-27.1
Scale reduction (Central Meridian)	1 : 160,000
$\log \left(\frac{1}{6\rho_0^2} \right)_g$	4.580 8361 -20
$\log \left(\frac{1}{6\rho_0^2 \sin l''} \right)_g$	9.895 2612 -20
$\left(\frac{1}{6\rho_0^2 \sin l''} \right)_g$	0.7857×10^{-10}

TRANSVERSE MERCATOR PROJECTION
RHODE ISLAND

Lat.	y. feet	$\Delta y.$ per second	H	ΔH per second	V	ΔV per second	a
41 05	0 00	101.207 50	76.581 572	322.53	1.220 084	1.65	- .979
41 06	6 072.45	101.207 83	76.562 220	322.65	1.220 183	1.63	- .974
41 07	12 144.92	101.208 17	76.542 861	322.76	1.220 281	1.63	- .970
41 08	18 217.41	101.208 50	76.523 495	322.86	1.220 379	1.63	- .966
41 09	24 289.92	101.208 83	76.504 123	322.98	1.220 477	1.61	- .961
41 10	30 362.45	101.209 00	76.484 744	323.08	1.220 574	1.60	- .957
41 11	36 434.99	101.209 33	76.465 359	323.20	1.220 670	1.61	- .953
41 12	42 507.55	101.209 67	76.445 967	323.30	1.220 767	1.58	- .949
41 13	48 580.13	101.209 83	76.426 569	323.41	1.220 862	1.58	- .944
41 14	54 652.72	101.210 33	76.407 164	323.51	1.220 957	1.58	- .940
41 15	60 725.34	101.210 50	76.387 753	323.63	1.221 052	1.58	- .936
41 16	66 797.97	101.210 83	76.368 335	323.73	1.221 147	1.56	- .932
41 17	72 870.62	101.211 17	76.348 911	323.85	1.221 241	1.55	- .928
41 18	78 943.29	101.211 33	76.329 480	323.95	1.221 334	1.55	- .923
41 19	85 015.97	101.211 67	76.310 043	324.06	1.221 427	1.55	- .919
41 20	91 088.67	101.212 00	76.290 599	324.16	1.221 520	1.53	- .915
41 21	97 161.39	101.212 33	76.271 149	324.28	1.221 612	1.53	- .911
41 22	103 234.13	101.212 67	76.251 692	324.38	1.221 704	1.51	- .906
41 23	109 306.89	101.212 83	76.232 229	324.50	1.221 795	1.51	- .902
41 24	115 379.66	101.213 33	76.212 759	324.60	1.221 886	1.51	- .898
41 25	121 452.46	101.213 50	76.193 283	324.71	1.221 977	1.50	- .894
41 26	127 525.27	101.213 83	76.173 800	324.81	1.222 067	1.50	- .889
41 27	133 598.10	101.214 00	76.154 311	324.93	1.222 157	1.48	- .885
41 28	139 670.94	101.214 33	76.134 815	325.03	1.222 246	1.48	- .881
41 29	145 743.80	101.214 83	76.115 313	325.15	1.222 335	1.48	- .876
41 30	151 816.69	101.215 00	76.095 804	325.25	1.222 424	1.46	- .872
41 31	157 889.59	101.215 17	76.076 289	325.36	1.222 512	1.46	- .868
41 32	163 962.50	101.215 67	76.056 767	325.46	1.222 600	1.45	- .864
41 33	170 035.44	101.215 83	76.037 239	325.58	1.222 687	1.45	- .859
41 34	176 108.39	101.216 17	76.017 704	325.68	1.222 774	1.45	- .855
41 35	182 181.36	101.216 50	75.998 163	325.80	1.222 861	1.43	- .851
41 36	188 254.35	101.216 83	75.978 615	325.88	1.222 947	1.43	- .847
41 37	194 327.36	101.217 00	75.959 062	326.01	1.223 033	1.41	- .843
41 38	200 400.38	101.217 33	75.939 501	326.11	1.223 118	1.41	- .838
41 39	206 473.42	101.217 67	75.919 934	326.21	1.223 203	1.41	- .834
41 40	212 546.48	101.218 00	75.900 361	326.33	1.223 288	1.40	- .830
41 41	218 619.56	101.218 33	75.880 781	326.43	1.223 372	1.40	- .826
41 42	224 692.66	101.218 50	75.861 195	326.55	1.223 456	1.38	- .822
41 43	230 765.77	101.218 83	75.841 602	326.65	1.223 539	1.38	- .817
41 44	236 838.90	101.219 17	75.822 003	326.75	1.223 622	1.38	- .813
41 45	242 912.05	101.219 50	75.802 398	326.86	1.223 705	1.36	- .809
41 46	248 985.22	101.219 67	75.782 786	326.96	1.223 787	1.36	- .805
41 47	255 058.40	101.220 17	75.763 168	327.08	1.223 869	1.35	- .801
41 48	261 131.61	101.220 33	75.743 543	327.18	1.223 950	1.35	- .796
41 49	267 204.83	101.220 67	75.723 912	327.30	1.224 031	1.35	- .792
41 50	273 278.07	101.220 83	75.704 274	327.40	1.224 112	1.33	- .788
41 51	279 351.32	101.221 33	75.684 630	327.50	1.224 192	1.33	- .784
41 52	285 424.60	101.221 50	75.664 980	327.61	1.224 272	1.33	- .780
41 53	291 497.89	101.221 83	75.645 323	327.73	1.224 352	1.31	- .775
41 54	297 571.20	101.222 17	75.625 659	327.81	1.224 431	1.30	- .771
41 55	303 644.53	101.222 50	75.605 990	327.95	1.224 509	1.30	- .767
41 56	309 717.88	101.222 67	75.586 313	328.03	1.224 587	1.30	- .763
41 57	315 791.24	101.223 00	75.566 631	328.15	1.224 665	1.30	- .759
41 58	321 864.62	101.223 33	75.546 942	328.26	1.224 743	1.26	- .754
41 59	327 938.02	101.223 67	75.527 246	328.36	1.224 819	1.28	- .750
42 00	334 011.44		75.507 544		1.224 896		- .746

TRANSVERSE MERCATOR PROJECTION
RHODE ISLAND

Lat.		y _o feet	Δy _o per second	H	ΔH per second	V	ΔV per second	a
42 0 0		334 011 .44	101.224 00	75.507 544	328.48	1.224 896	1.26	-.746
42 0 1		340 084 .88	101.224 17	75.487 835	328.56	1.224 972	1.26	-.742
42 0 2		346 158 .33	101.224 50	75.468 121	328.70	1.225 048	1.25	-.738
42 0 3		352 231 .80	101.224 83	75.448 399	328.80	1.225 123	1.25	-.733
42 0 4		358 305 .29	101.225 17	75.428 671	328.90	1.225 198	1.23	-.729
42 0 5		364 378 .80	101.225 33	75.408 937	329.00	1.225 272	1.23	-.725
42 0 6		370 452 .32	101.225 83	75.389 197	329.11	1.225 346	1.23	-.721
42 0 7		376 525 .87	101.226 00	75.369 450	329.23	1.225 420	1.21	-.717
42 0 8		382 599 .43	101.226 33	75.349 696	329.33	1.225 493	1.21	-.712
42 0 9		388 673 .01	101.226 50	75.329 936	329.43	1.225 566	1.20	-.708
42 1 0		394 746 .60	101.227 00	75.310 170	329.55	1.225 638	1.20	-.704
42 1 1		400 820 .22	101.227 17	75.290 397	329.65	1.225 710	1.20	-.700
42 1 2		406 893 .85	101.227 50	75.270 618	329.76	1.225 782	1.18	-.696
42 1 3		412 967 .50	101.227 83	75.250 832	329.86	1.225 853	1.18	-.691
42 1 4		418 041 .17	101.228 17	75.231 040	329.96	1.225 924	1.16	-.687
42 1 5		425 114 .86		75.211 242		1.225 994		-.683

TRANSVERSE MERCATOR PROJECTION

RHODE ISLAND

$\Delta \lambda''$	b	Δb	c	$\Delta \lambda''$	b	Δb	c
0	0.000	+0.068	0.000	2100	+1.025	+0.009	-0.052
100	+0.068	+0.067	0.000	2200	+1.034	+0.004	-0.055
200	+0.135	+0.067	-0.001	2300	+1.038	-0.002	-0.057
300	+0.202	+0.066	-0.002	2400	+1.036	-0.008	-0.059
400	+0.268	+0.065	-0.003	2500	+1.028	-0.014	-0.060
500	+0.333	+0.063	-0.004	2600	+1.014	-0.022	-0.061
600	+0.396	+0.062	-0.006	2700	+0.992	-0.028	-0.062
700	+0.458	+0.060	-0.008	2800	+0.964	-0.034	-0.062
800	+0.518	+0.058	-0.010	2900	+0.930	-0.043	-0.063
900	+0.576	+0.056	-0.012	3000	+0.887	-0.050	-0.062
1000	+0.632	+0.053	-0.015	3100	+0.837	-0.058	-0.061
1100	+0.685	+0.050	-0.018	3200	+0.779	-0.067	-0.059
1200	+0.735	+0.047	-0.021	3300	+0.712	-0.074	-0.056
1300	+0.782	+0.044	-0.024	3400	+0.638	-0.083	-0.052
1400	+0.826	+0.041	-0.028	3500	+0.555	-0.093	-0.047
1500	+0.867	+0.037	-0.032	3600	+0.462	-0.101	-0.040
1600	+0.904	+0.033	-0.035	3700	+0.361	-0.110	-0.032
1700	+0.937	+0.029	-0.038	3800	+0.251	-0.121	-0.023
1800	+0.966	+0.024	-0.041	3900	+0.130	-0.130	-0.012
1900	+0.990	+0.020	-0.045	4000	0.000		0.000
2000	+1.010	+0.015	-0.049				

TRANSVERSE MERCATOR PROJECTION

TABLE FOR g

$$\Delta \alpha'' = \sin \phi (\Delta \lambda'') + g$$

Latitude	$\Delta \lambda''$						
	0"	1000"	2000"	3000"	4000"	5000"	6000"
24°	0.00	0.00	0.02	0.07	0.17	0.33	0.58
25	0	0	0.02	0.07	0.17	0.34	0.59
26°	0.00	0.00	0.02	0.08	0.18	0.35	0.60
27	0	0	0.02	0.08	0.18	0.35	0.61
28	0	0	0.02	0.08	0.18	0.36	0.62
29	0	0	0.02	0.08	0.19	0.37	0.63
30	0	0	0.02	0.08	0.19	0.37	0.64
31°	0.00	0.00	0.02	0.08	0.19	0.37	0.64
32	0	0	0.02	0.08	0.19	0.38	0.65
33	0	0	0.02	0.08	0.19	0.38	0.65
34	0	0	0.02	0.08	0.19	0.38	0.65
35	0	0	0.02	0.08	0.19	0.38	0.65
36°	0.00	0.00	0.02	0.08	0.19	0.38	0.65
37	0	0	0.02	0.08	0.19	0.38	0.65
38	0	0	0.02	0.08	0.19	0.38	0.65
39	0	0	0.02	0.08	0.19	0.37	0.64
40	0	0	0.02	0.08	0.19	0.37	0.64
41°	0.00	0.00	0.02	0.08	0.19	0.37	0.63
42	0	0	0.02	0.08	0.18	0.36	0.63
43	0	0	0.02	0.08	0.18	0.36	0.62
44	0	0	0.02	0.08	0.18	0.35	0.61
45	0	0	0.02	0.08	0.18	0.35	0.60
46°	0.00	0.00	0.02	0.07	0.17	0.34	0.59
47	0	0	0.02	0.07	0.17	0.33	0.58
48	0	0	0.02	0.07	0.17	0.33	0.56
49	0	0	0.02	0.07	0.16	0.32	0.55
50	0.00	0.00	0.02	0.07	0.16	0.31	0.54

$$g = \left[\frac{C (\sin l'') \cos^3 \phi}{2A^2} + F \right] (\Delta \lambda'')^3$$

A, C and F are position factors.

Y CORRECTION FOR COMPUTATION OF GEOGRAPHIC
POSITIONS FROM PLANE COORDINATES
TRANSVERSE MERCATOR PROJECTION, RHODE ISLAND

$$P(x'/10,000)^2 + d = V(\Delta y 100)^2 + c$$

P taken out for y-coordinate
d taken out for x'

y	P	ΔP	x'	d
0	2.08005	2016	0	0.00
100,000	2.10021	2034	50,000	0.00
200,000	2.12055	2051	100,000	0.00
300,000	2.14106	2069	150,000	+ 0.01
400,000	2.16175		200,000	0.00
			220,000	0.00

TRANSVERSE MERCATOR PROJECTION

Rhode Island

$$\Delta\alpha = Mx' - e$$

y	M	ΔM
0	0.008 5811	832
100,000	0.008 6643	839
200,000	0.008 7482	846
300,000	0.008 8328	854
400,000	0.008 9182	

x'	e
100,000	0.0
200,000	0.1

TRANSVERSE MERCATOR PROJECTION FOR RHODE ISLAND

x' (feet)	Scale in units of 7th place of logs	Scale ex- pressed as a ratio	x' (feet)	Scale in units of 7th place of logs	Scale ex- pressed as a ratio
0	- 27.1	0.9999938	175,000	+124.9	1.0000288
5,000	- 27.0	0.9999938	180,000	+133.8	1.0000308
10,000	- 26.6	0.9999939	185,000	+142.8	1.0000329
15,000	- 26.0	0.9999940	190,000	+152.1	1.0000350
20,000	- 25.1	0.9999942	195,000	+161.7	1.0000372
25,000	- 24.0	0.9999945	200,000	+171.5	1.0000395
30,000	- 22.6	0.9999948	205,000	+181.5	1.0000418
35,000	- 21.0	0.9999952	210,000	+191.9	1.0000442
40,000	- 19.2	0.9999956	215,000	+202.4	1.0000466
45,000	- 17.0	0.9999961	220,000	+213.2	1.0000491
50,000	- 14.7	0.9999966			
55,000	- 12.1	0.9999972			
60,000	- 9.2	0.9999979			
65,000	- 6.1	0.9999986			
70,000	- 2.8	0.9999994			
75,000	+ 0.8	1.0000002			
80,000	+ 4.7	1.0000011			
85,000	+ 8.8	1.0000020			
90,000	+ 13.1	1.0000030			
95,000	+ 17.7	1.0000041			
100,000	+ 22.5	1.0000052			
105,000	+ 27.6	1.0000064			
110,000	+ 33.0	1.0000076			
115,000	+ 38.6	1.0000089			
120,000	+ 44.4	1.0000102			
125,000	+ 50.5	1.0000116			
130,000	+ 56.8	1.0000131			
135,000	+ 63.4	1.0000146			
140,000	+ 70.2	1.0000162			
145,000	+ 77.3	1.0000178			
150,000	+ 84.6	1.0000195			
155,000	+ 92.2	1.0000212			
160,000	+100.0	1.0000230			
165,000	+108.1	1.0000249			
170,000	+116.4	1.0000268			

CORRECTIONS TO NATURAL SCALE RATIOS*

(in units of the 7th decimal place)

For Lambert Projection				For Lambert or transverse Mercator Projection	
<u>$\Delta\phi'$</u> as argument				<u>Δy</u> or <u>Δx</u>	Corr'n (Plus)
<u>$\Delta\phi'$</u>	Corr'n (Plus)	<u>$\Delta\phi'$</u>	Corr'n (Plus)		
1	0	31	34	10,000	0
2	0	32	36	20,000	0
3	0	33	38	30,000	1
4	1	34	40	40,000	2
5	1	35	43	50,000	2
6	1	36	45	60,000	3
7	2	37	48	70,000	5
8	2	38	51	80,000	6
9	3	39	53	90,000	8
10	4	40	56	100,000	10
11	4	41	59	110,000	11
12	5	42	62	120,000	14
13	6	43	65	130,000	16
14	7	44	68	140,000	19
15	8	45	71	150,000	21
16	9	46	74	160,000	24
17	10	47	77	170,000	27
18	11	48	81	180,000	31
19	13	49	84	190,000	34
20	14	50	88	200,000	38
21	15	51	91	210,000	42
22	17	52	95	220,000	46
23	19	53	98	230,000	50
24	20	54	102	240,000	55
25	22	55	106	250,000	59
26	24	56	110	260,000	64
27	26	57	114	270,000	69
28	27	58	118	280,000	74
29	29	59	122	290,000	80
30	32	60	126	300,000	86
$\Delta\phi'$ is the difference in latitude in minutes of the ends of the line.				310,000	91
				320,000	97
				330,000	103
				340,000	110
				350,000	116

*Scale ratio interpolated for mean latitude or mean x' of the ends of a line and corrected by the above table is a true mean value accurate to within one in the seventh decimal place.