

NOAA Technical Memorandum NOS NGS 17



THE HAVAGO THREE-DIMENSIONAL ADJUSTMENT PROGRAM

T. Vincenty

Rockville, Md.
May 1979

NOAA Technical Publications

National Ocean Survey/National Geodetic Survey subseries

The National Geodetic Survey (NGS) of the National Ocean Survey (NOS), NOAA, establishes and maintains the basic National horizontal and vertical networks of geodetic control and provides governmentwide leadership in the improvement of geodetic surveying methods and instrumentation, coordinates operations to assure network development, and provides specifications and criteria for survey operations by Federal, State, and other agencies.

NGS engages in research and development for the improvement of knowledge of the figure of the Earth and its gravity field, and has the responsibility to procure geodetic data from all sources, process these data, and make them generally available to users through a central data base.

NOAA Technical Memorandums and some special NOAA publications are sold by the National Technical Information Service (NTIS) in paper copy and microfiche. Orders should be directed to NTIS, 5285 Port Royal Road, Springfield, VA 22161 (telephone: 703-557-4650). NTIS customer charge accounts are invited; some commercial charge accounts are accepted. When ordering, give the NTIS accession number (which begins with PB) shown in parentheses in the following citations.

Paper copies of NOAA Technical Reports, which are of general interest to the public, are sold by the Superintendent of Documents, U.S. Government Printing Office (GPO), Washington, DC 20402 (telephone: 202-783-3238). For prompt service, please furnish the GPO stock number with your order. If a citation does not carry this number, then the publication is not sold by GPO. All NOAA Technical Reports may be purchased from NTIS in hard copy and microform. Prices for the same publication may vary between the two Government sales agents. Although both are nonprofit, GPO relies on some Federal support whereas NTIS is self-sustained.

An excellent reference source for Government publications is the National Depository Library program, a network of about 1,300 designated libraries. Requests for borrowing Depository Library material may be made through your local library. A free listing of libraries currently in this system is available from the Library Division, U.S. Government Printing Office, 5236 Eisenhower Ave., Alexandria, VA 22304 (telephone: 703-557-9013).

NOAA geodetic publications

Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1974 reprinted annually, 12 pp (PB265442). National specifications and tables show the closures required and tolerances permitted for first-, second-, and third-order geodetic control surveys.

Specifications To Support Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1975, reprinted annually 30 pp (PB261037). This publication provides the rationale behind the original publication, "Classification, Standards of Accuracy, ..." cited above.

NOAA Technical Memorandums, NOS/NGS subseries

- NOS NGS-1 Use of climatological and meteorological data in the planning and execution of National Geodetic Survey field operations. Robert J. Leffler, December 1975, 30 pp (PB249677). Availability, pertinence, uses, and procedures for using climatological and meteorological data are discussed as applicable to NGS field operations.
- NOS NGS-2 Final report on responses to geodetic data questionnaire. John F. Spencer, Jr., March 1976, 39 pp (PB254641). Responses (20%) to a geodetic data questionnaire, mailed to 36,000 U.S. land surveyors, are analyzed for projecting future geodetic data needs.
- NOS NGS-3 Adjustment of geodetic field data using a sequential method. Marvin C. Whiting and Allen J. Pope, March 1976, 11 pp (PB253967). A sequential adjustment is adopted for use by NGS field parties.
- NOS NGS-4 Reducing the profile of sparse symmetric matrices. Richard A. Snay, June 1976, 24 pp (PB-258476). An algorithm for improving the profile of a sparse symmetric matrix is introduced and tested against the widely used reverse Cuthill-McKee algorithm.
- NOS NGS-5 National Geodetic Survey data: availability, explanation, and application. Joseph F. Dracup, June 1976, 45 pp (PB258475). The summary gives data and services available from NGS, accuracy of surveys, and uses of specific data.

(Continued at end of publication)

NOAA Technical Memorandum NOS NGS 17

THE HAVAGO THREE-DIMENSIONAL ADJUSTMENT PROGRAM

T. Vincenty

Rockville, Md.
May 1979

UNITED STATES
DEPARTMENT OF COMMERCE
Juanita M. Kreps, Secretary

NATIONAL OCEANIC AND
ATMOSPHERIC ADMINISTRATION
Richard A. Frank, Administrator

National Ocean
Survey
Allen L. Powell, Director



CONTENTS

| | |
|--|----|
| Abstract | 1 |
| 1. Introduction | 1 |
| 2. Symbols and definitions | 3 |
| 3. General equations | 5 |
| 4. Observation equations | 6 |
| 5. Treatment of refraction | 10 |
| 6. Relative distances | 11 |
| 7. Adjustment with invariable astronomic coordinates.. | 11 |
| 8. Weights | 14 |
| 9. Error analysis | 14 |
| 10. Diagnostic messages | 16 |
| 11. Miscellaneous computations | 17 |
| References | 18 |

Mention of a commercial company or product does not constitute an endorsement by NOAA National Ocean Survey. Use for publicity or advertising purposes of information from this publication concerning proprietary products or the tests of such products is not authorized.

THE HAVAGO THREE-DIMENSIONAL ADJUSTMENT PROGRAM

T. Vincenty
National Geodetic Survey
National Ocean Survey, NOAA
Rockville, Md. 20852

ABSTRACT. HAVAGO is a computer program for adjusting numerous kinds of geodetic observations in three dimensions. It is not intended for handling very large networks, but is well suited for special surveys of highest precision and often with unusual configurations, as well as for any ordinary survey project. This publication gives a description of the program and the mathematical formulas on which it is founded.

1. INTRODUCTION

HAVAGO, an acronym for Horizontal And Vertical Adjustment of Geodetic Observations, is a computer program for adjusting 18 kinds of observations by the method of variation of parameters in three dimensions. It was designed for speedy and convenient handling of adjustments and analyses of special high precision surveys of the National Ocean Survey's National Geodetic Survey. The first operational version of the program was written in FORTRAN and implemented on the NOAA IBM 360/195 system by the author in November 1977, with an adjustment subroutine package prepared by William H. Dillinger. This publication contains a description of the main features of the program and gives the relevant mathematical equations.

The program accepts as observations horizontal (unoriented) directions, astronomic azimuths, reciprocal vertical angles, grouped vertical angles, orthometric height differences, spatial distances, relative (unscaled) distances, plane (horizontal) distances, position differences, differences in astronomic latitudes and longitudes, constrained positional components, and constrained astronomic latitudes and longitudes.

In the design of the program certain problems were given special attention:

- The program was structured so that it can handle any project, regardless of accuracy and the kinds of observations of which it is composed.

- Distances measured in the relative (ratio) mode have been introduced as a special class of observations. Each

group of such distances receives a scale unknown which is determined with respect to a supplied source of scale. Moreover, the program can determine the average scale for the project from relative distances alone.

- The handling of refraction corrections to vertical angles has been made flexible. The refraction model considers not only the mean coefficient of refraction over a line but also its change with height.

- The troublesome reductions of distances and vertical angles to the marks before the adjustment and between iterations have been avoided by using as observations the original measurements between the instrument and the target. This approach is also used by Sikonia (1977).

- Certain configurations which are not encountered in ordinary control surveys but are common in special surveys have been anticipated. For example, there may be a point on a tower and another one on the ground and not necessarily plumbed.

In principle, an ellipsoid is not needed for performing an adjustment in three dimensions, but it has been retained in HAVAGO for convenience. One reason for this is that a parametric adjustment requires the input of provisional positions of stations and these are normally known in a geographic coordinate system. However, observations are not reduced to the ellipsoid, nor are any computations carried out on it, except after the adjustment to obtain geodetic azimuths and distances over selected lines. The differential shifts of coordinates are in linear units in a rectangular horizon system centered on the point in question; thus, there are as many local coordinate systems as there are stations. This is a convenient arrangement, making it possible to choose preliminary astronomic coordinates to be the same as geodetic values on an arbitrary reference ellipsoid, as shown in section 4.

Another reason for using ellipsoidal coordinates is that observation equations for orthometric height differences cannot be formed without them. This class of observations is used in the program with the understanding that it is admissible only under the same assumptions as in classical geodesy.

As now written, the program can handle up to 250 stations and 1,250 unknowns, and requires 186K bytes of core.

2. SYMBOLS AND DEFINITIONS

| | |
|----------------------|---|
| a, b | equatorial radius and minor semiaxis of the ellipsoid |
| e | first eccentricity of the ellipsoid |
| ϵ | the square of second eccentricity |
| M | radius of curvature in the meridian |
| N | radius of curvature in the prime vertical |
| ϕ | geodetic latitude, positive north |
| λ | geodetic longitude, positive east |
| h | geodetic height |
| ϕ' , λ' | astronomic latitude and longitude, ground level values |
| H | height above geoid |
| A | astronomic azimuth, clockwise from north |
| V | vertical angle, positive upwards from astronomic horizon |
| S | spatial distance |
| X, Y, Z | Cartesian coordinates in the equatorial system, with X-axis passing through the Greenwich meridian and Z-axis parallel to the mean rotation axis of the Earth |
| dx, dy, dh | differential shifts of coordinates of a point in a local horizon system, positive north, east, and upwards, respectively |
| P, Q, T | Cartesian coordinates of the forepoint (north, east, upwards) in a local astronomic horizon system centered on the standpoint |

Subscripts 1 and 2 denote the standpoint and the forepoint, respectively. Subscript 0 denotes observed values.

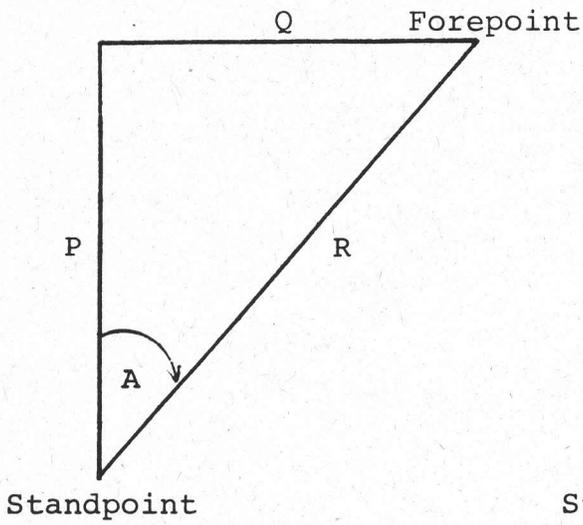


Figure 1.--Top view.

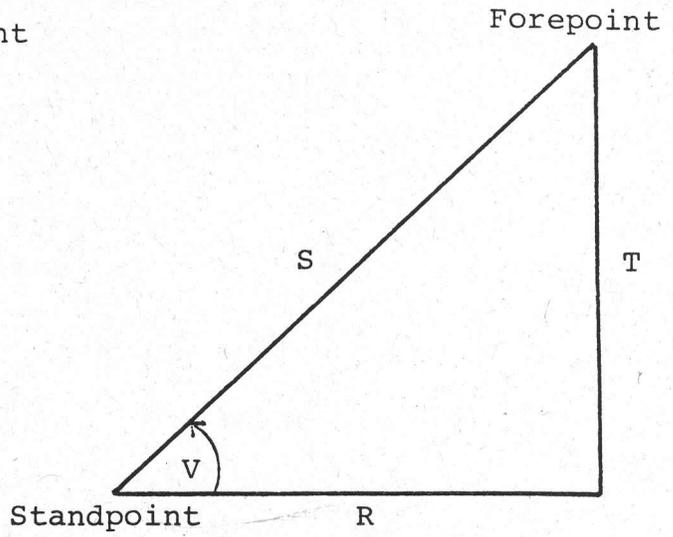


Figure 2.--Side view.

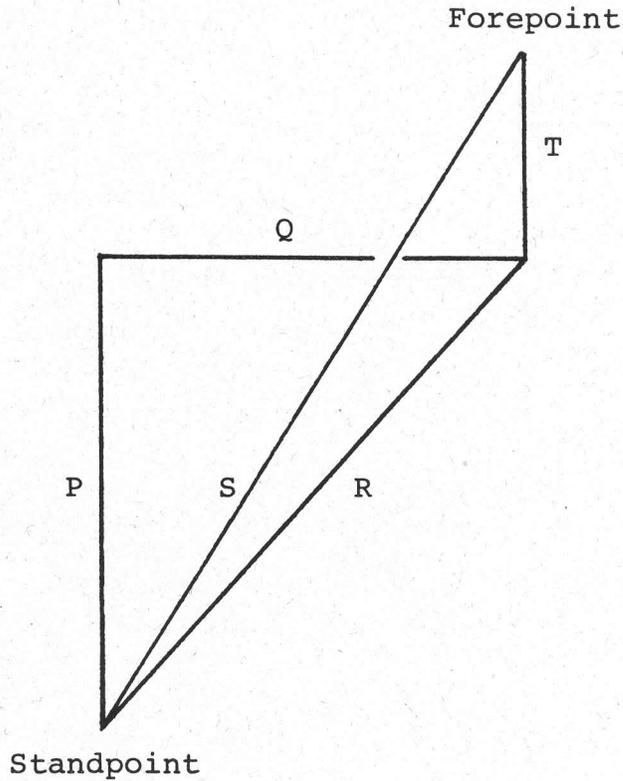


Figure 3.--Perspective view.

3. GENERAL EQUATIONS

a. Conversion of ϕ , λ , h to X , Y , Z .

$$X = (N + h) \cos \phi \cos \lambda.$$

$$Y = (N + h) \cos \phi \sin \lambda.$$

$$Z = [N(1 - e^2) + h] \sin \phi. \quad (3.1)$$

b. Inverse formula in space. See figures 1, 2, and 3.

$$\Delta X = X_2 - X_1 \quad \Delta Y = Y_2 - Y_1 \quad \Delta Z = Z_2 - Z_1. \quad (3.2)$$

$$P = -\sin \phi'_1 (\Delta X \cos \lambda'_1 + \Delta Y \sin \lambda'_1) + \Delta Z \cos \phi'_1 \quad (3.3)$$

$$Q_1 = -\Delta X \sin \lambda'_1 + \Delta Y \cos \lambda'_1. \quad (3.4)$$

$$R_1 = \sqrt{P_1^2 + Q_1^2}. \quad (3.5)$$

$$T_1 = \cos \phi'_1 (\Delta X \cos \lambda'_1 + \Delta Y \sin \lambda'_1) + \Delta Z \sin \phi'_1. \quad (3.6)$$

$$S = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}. \quad (3.7)$$

$$\tan A_{12} = Q_1/P_1. \quad (3.8)$$

$$\tan V_{12} = T_1/R_1. \quad (3.9)$$

For the inverse direction we have

$$P_2 = \sin \phi'_2 (\Delta X \cos \lambda'_2 + \Delta Y \sin \lambda'_2) - \Delta Z \cos \phi'_2. \quad (3.10)$$

$$Q_2 = \Delta X \sin \lambda'_2 - \Delta Y \cos \lambda'_2. \quad (3.11)$$

$$R_2 = \sqrt{P_2^2 + Q_2^2}. \quad (3.12)$$

$$T_2 = -\cos \phi'_2 (\Delta X \cos \lambda'_2 + \Delta Y \sin \lambda'_2) - \Delta Z \sin \phi'_2. \quad (3.13)$$

$$\tan A_{21} = Q_2/P_2. \quad (3.14)$$

$$\tan V_{21} = T_2/R_2. \quad (3.15)$$

c. Conversion of X, Y, Z to ϕ , λ , h.

$$p = \sqrt{X^2 + Y^2}. \quad (3.16)$$

$$\tan \theta = (Z/p)(a/b). \quad (3.17)$$

$$\tan \phi = \frac{Z + \epsilon b \sin^3 \theta}{p - e^2 a \cos^3 \theta}. \quad (3.18)$$

$$\tan \lambda = Y/X. \quad (3.19)$$

$$\tan u = (b/a) \tan \phi. \quad (3.20)$$

$$h = \sqrt{(p - a \cos u)^2 + (Z - b \sin u)^2}. \quad (3.21)$$

The sign of h is the same as the sign of $(p - a \cos u)$.

The latitude formula is that of Bowring (1976). The height formula is given in different forms by Bartelme and Meissl (1975), and by Bopp and Krauss (1976).

d. Transformation of coordinate shifts by rotation of axes.

Let

$$R_0 = \begin{bmatrix} -\sin \phi \cos \lambda & -\sin \phi \sin \lambda & \cos \phi \\ -\sin \lambda & \cos \lambda & 0 \\ \cos \phi \cos \lambda & \cos \phi \sin \lambda & \sin \phi \end{bmatrix}. \quad (3.22)$$

Then

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = R_0^T \begin{bmatrix} dx \\ dy \\ dh \end{bmatrix} \quad (3.23)$$

$$\begin{bmatrix} dx \\ dy \\ dh \end{bmatrix} = R_0 \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}. \quad (3.24)$$

e. Conversion of rectangular coordinate shifts to geographic equivalents.

$$d\phi = dx/(M + h) \quad d\lambda = dy/[(N + h) \cos \phi]. \quad (3.25)$$

4. OBSERVATION EQUATIONS

The observation equations for astronomic azimuth, spatial distance, and astronomic vertical angle are respectively

$$v_A = a_1 dx_1 + a_2 dy_1 + a_3 dh_1 + a_4 dx_2 + a_5 dy_2 + a_6 dh_2 \\ + a_7 d\phi'_1 + a_8 d\lambda'_1 + K_A \quad (4.1)$$

$$v_S = b_1 dx_1 + b_2 dy_1 + b_3 dh_1 + b_4 dx_2 + b_5 dy_2 + b_6 dh_2 + K_S \quad (4.2)$$

$$v_V = c_1 dx_1 + c_2 dy_1 + c_3 dh_1 + c_4 dx_2 + c_5 dy_2 + c_6 dh_2 \\ + c_7 d\phi'_1 + c_8 d\lambda'_1 + K_V. \quad (4.3)$$

The constant terms K_A , K_S , K_V are the differences between the assumed (i.e., computed from provisional positions) and observed values. The unknowns are differential shifts of position components and of astronomic latitude and longitude. In addition, there are orientation unknowns for horizontal directions, refraction unknowns for vertical angles, and scale unknowns for relative distances, as will be shown later.

In forming the coefficients and the constant terms of observation equations, the preliminary astronomic latitudes and longitudes are set to the corresponding most recent geodetic values. This is permitted by theory (Wolf 1963, p. 231) and, from a practical point of view, very convenient. It is then unnecessary to keep in core the arrays of astronomic values and to update them between iterations. It is quite obvious that the distinction between astronomic and geodetic values is immaterial for this purpose. If the direction of the vertical is changed by 20", this results in a change in adjusted position of only 1 micron per centimeter of positional shift. Also, if the constant term is changed by 20" by adoption of false astronomic values, this change is well within differential range, because arc 20" and $\sin 20''$ have the same values to six significant figures.

The coefficients of eqs. (4) are given in simplified forms after generalizing those of a previous paper (Vincenty and Bowring 1978). They are

$$a_1 = Q_1/R_1^2$$

$$a_2 = -P_1/R_1^2$$

$$a_3 = 0$$

$$a_4 = -[Q_1(\cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 \cos \Delta\lambda) \\ + P_1 \sin \phi_2 \sin \Delta\lambda]/R_1^2$$

$$a_5 = (P_1 \cos \Delta\lambda - Q_1 \sin \phi_1 \sin \Delta\lambda)/R_1^2$$

$$a_6 = 0$$

$$a_7 = Q_1 T_1/R_1^2$$

$$a_8 = \sin \phi_1 - \cos \phi_1 P_1 T_1/R_1^2$$

$$b_1 = - P_1/S$$

$$b_2 = - Q_1/S$$

$$b_3 = - T_1/S$$

$$b_4 = - P_2/S$$

$$b_5 = - Q_2/S$$

$$b_6 = - T_2/S$$

$$c_1 = P_1 T_1 / (R_1 S^2)$$

$$c_2 = Q_1 T_1 / (R_1 S^2)$$

$$c_3 = - R_1 / S^2$$

$$c_4 = (- \cos \phi_1 \sin \phi_2 \cos \Delta \lambda + \sin \phi_1 \cos \phi_2 + T_1 P_2 / S^2) / R_1$$

$$c_5 = (- \cos \phi_1 \sin \Delta \lambda + T_1 Q_2 / S^2) / R_1$$

$$c_6 = (\cos \phi_1 \cos \phi_2 \cos \Delta \lambda + \sin \phi_1 \sin \phi_2 + T_1 T_2 / S^2) / R_1$$

$$c_7 = P_1 / R_1$$

$$c_8 = \cos \phi_1 Q_1 / R_1.$$

The coefficient a_6 is given more precisely by

$$a_6 = [Q_1 (\sin \phi_1 \cos \phi_2 - \cos \phi_1 \sin \phi_2) + P_1 \cos \phi_2 \sin \Delta \lambda] / R_1^2$$

(Heiskanen and Moritz 1967, Mitchell 1963, Rapp 1975, Wolf 1963). However, for a shift of the forepoint by 1 meter in height, the approximation $a_6 = 0$ can introduce a maximum error of one unit in the fourth decimal of a second of direction or azimuth. In practice this error will be even smaller, since adjustments are iterated until the last shifts are in centimeters or in millimeters; therefore, this coefficient was set to zero.

Equation (4.1) is also used for horizontal unoriented directions with the addition of an orientation unknown with a coefficient of -1.

The observations are not reduced to the marks on the ground; rather, they refer to the line in space between the instrument and the target at the time of observing. It is inconvenient to reduce observations to the marks for two reasons. Such a reduction, performed before the adjustment, should not be accepted as final because its accuracy depends on the accuracies of the heights which are subject to change. Also, if the

reductions are accomplished before each iteration, this may slow down the convergence of the adjustment. After the adjustment the reduction to the marks is obtained in an indirect way by applying the residuals, refraction corrections, and scale corrections (with signs reversed) to the values computed by the inverse formula from adjusted coordinates of the marked point, thus obtaining the values that would have been observed if the heights of the instrument and the target were zero.

The use of orthometric height differences as observations is an approximation to the extent that an assumption is made that the direction of the gravity vector varies uniformly between the stations. Under this assumption the observation equation is

$$v_{\Delta H} = -dh_1 + dh_2 + (P_1/2) d\phi'_1 + (Q_1 \cos \phi_1 / 2) d\lambda'_1 \\ - (P_2/2) d\phi'_2 - (Q_2 \cos \phi_2 / 2) d\lambda'_2 + \Delta h - \Delta H, \quad (4.4)$$

where Δh is the assumed difference in geodetic height and ΔH is the observed orthometric height difference. By strict theory, the constant term should be modified to account for the effects of the curvature of the plumb line; however, this is ignored because the curvature will generally be unknown, and even corrections based on normal gravity are believed to be less certain than the assumption of a uniform change in deflections.

For geodetic position differences between nearby points we have the equations

$$v_{\Delta x} = -dx_1 + dx_2 + (\phi_2 - \phi_1)M_1 - \Delta x_0 \\ v_{\Delta y} = -dy_1 + dy_2 + (\lambda_2 - \lambda_1)N_1 \cos \phi_1 - \Delta y_0 \\ v_{\Delta h} = -dh_1 + dh_2 + h_2 - h_1 - \Delta h_0, \quad (4.5)$$

where ϕ , λ , and h are the most recent values.

The observation equation for a short plane distance s is

$$v_s = -\Delta x dx_1 - \Delta y dy_1 + \Delta x dx_2 + \Delta y dy_2 \\ + \sqrt{\Delta x^2 + \Delta y^2} - s_0, \quad (4.6)$$

in which

$$\Delta x = (\phi_2 - \phi_1)(M_1 + h_1) \\ \Delta y = (\lambda_2 - \lambda_1)(N_1 + h_1) \cos \phi_1.$$

Astronomic coordinate differences may be used as observations to connect several points in a small area where astronomic observations were made at only one station, and it can safely be assumed that these differences are the same as the corresponding geodetic values. The observation equations are

$$\begin{aligned}v_{\Delta\phi}' &= -(1/M_1) dx_1 + (1/M_1) dx_2 + 0 \\v_{\Delta\lambda}' &= -[1/(N_1 \cos \phi_1)] dy_1 + [1/(N_1 \cos \phi_1)] dy_2 + 0.\end{aligned}\quad (4.7)$$

Station coordinates are constrained by the equations

$$\begin{aligned}v_{dx} &= dx + (\phi - \phi_0)(M + h) \\v_{dy} &= dy + (\lambda - \lambda_0)(N + h) \cos \phi \\v_{dh} &= dh + h - h_0.\end{aligned}\quad (4.8)$$

Finally, astronomic latitudes and longitudes are constrained by

$$\begin{aligned}v_{\phi}' &= d\phi' + \phi - \phi'_0 \\v_{\lambda}' &= d\lambda' + \lambda - \lambda'_0.\end{aligned}\quad (4.9)$$

5. TREATMENT OF REFRACTION

In the case of reciprocal vertical angles a refraction unknown with a coefficient of $-R_1$ is assigned to each pair of observations. This assumes that the coefficient of refraction k is the same over the whole line. Generally, k varies with height; therefore a correction for this change is applied to observed values. A default value of $dk/dh = -0.00001$ is used unless a different value is supplied in the input. The corrections to the angles are

$$\begin{aligned}c_{12} &= -(dk/dh) R_1 \Delta h / (12a) \\c_{21} &= (dk/dh) R_1 \Delta h / (12a),\end{aligned}\quad (5.1)$$

where $\Delta h = h_2 - h_1$.

Grouped vertical angles are assumed to have been measured from a standpoint in succession over several lines, but this requirement can be overridden by a flag in the input, in which case several standpoints are allowed to appear in one group. A refraction unknown with a coefficient of $-R_1$ is assigned to each group, and the corrections for change of k with height are handled in the same way as in the case of reciprocal angles.

If the values k_1 and k_2 are known and furnished, a refraction correction is applied to the angle, using the formula

$$\text{correction} = -R_1 a(2k_1 + k_2)/6, \quad (5.2)$$

and the correction for Δk is not applied.

6. RELATIVE DISTANCES

Relative distances are defined here as those measured for the primary purpose of establishing the shapes of geodetic figures by line ratios, as distinguished from their use as sources of scale. Such distances are normally measured from a standpoint to several forepoints in succession and in a manner similar to observing horizontal directions. A group of relative distances is assigned a scale unknown with a coefficient of $-S$, just like a set of directions is assigned an orientation unknown.

The inclusion of relative distances is an important feature of HAVAGO, useful not only in geodetic surveys but also in surveys for detection of motions of the Earth's crust or dams.

A provision has been included in the program to determine the scale of the survey from relative distances alone or in combination with a supplied source of scale. An observation equation is added for this purpose to make the sum of scale unknowns equal to zero. The validity of this method of determination of scale depends on the arrangement of measurements in time (Carter and Vincenty 1978, Vincenty 1978).

7. ADJUSTMENT WITH INVARIABLE ASTRONOMIC COORDINATES

In contrast to the classical treatment, in three-dimensional geodesy the astronomic latitudes and longitudes need not be held fixed, but may be permitted to acquire corrections. They may even be synthesized at stations with no astronomic observations, most strongly by precise vertical angles. Astronomic azimuths contribute to the determination of astronomic longitudes to some extent but are ineffective in low latitudes. Even unoriented directions can be of value in recovering astrogeodetic deflections, but only over very steep lines.

When the vertical angles are imprecise and are introduced in an adjustment with a standard deviation of several seconds, they are too weak to affect astronomic coordinates determined with much better accuracy. Under these circumstances it is profitable to fix the astronomic parameters by omitting their unknowns. This option can be exercised in HAVAGO.

Under this option the astronomic coordinates and their standard deviations are still required as input for all

stations. If not known, the astronomic coordinates may be allowed to assume the geodetic values, which is also the method implicitly applied in classical geodesy in such cases. The advantage of this option is that it reduces the number of unknowns to a minimum of three per station.

The astronomic coefficients of observation equations for theodolite observations and orthometric height differences are now evaluated in the same way as in the general case. They are multiplied by the appropriate known values of $(\phi' - \phi)$ and $(\lambda' - \lambda)$ and the results are added to the constant term of the observation equation. This is fully equivalent to correcting observations for astrogeodetic deflections.

This will be illustrated on a hypothetical example. Let the preliminary geodetic positions be the same as adjusted positions, so that $dx = dy = dh = 0$; in other words, an additional adjustment is performed after convergence has been achieved. The geodetic and astronomic values (the latter to be held fixed) are

| | <u>Standpoint</u> | <u>Forepoint</u> |
|------------|-------------------|------------------|
| ϕ | 30°00'00.0000" | 30°21'00.0000" |
| λ | 0.0000 | 0 43 00.0000 |
| h | 500.000 m | 3000.000 m |
| ϕ' | 30°00'05.00" | |
| λ' | 0 00 05.00 | |

Equations (3.1) to (3.9) give the following data:

| | <u>A₁</u> | <u>S</u> | <u>V₁</u> |
|--|----------------------|-------------|----------------------|
| True values, using ϕ' , λ' as observed | 60°28'56.305" | 79244.880 m | 1°27'13.533" |
| Assumed values, using $\phi' = \phi$ and $\lambda' = \lambda$ | 60 28 53.749 | 79244.880 | 1 27 07.302 |

From appropriate equations in section 4 we have

$$a_7 = 0.02206 \qquad a_8 = 0.48918.$$

Now suppose that the astronomic azimuth was observed as 62°28'56.00". The constant term of the observation will be computed as follows:

| | | |
|--------------------|---------------|------------------------|
| computed azimuth | 62°28'53.749" | |
| - observed azimuth | -60 28 56.000 | |
| + $a_7 d\phi'$ | 0.110 | ($d\phi' = +5''$) |
| + $a_8 d\lambda'$ | 2.446 | ($d\lambda' = +5''$) |
| sum = v | 0.305 | |

If the assumed astronomic values are chosen so that they coincide with the observed values (Vincenty and Bowring 1978), $d\phi' = d\lambda' = 0$ and we have

| | |
|--------------------|---------------|
| computed azimuth | 60°28'56.305" |
| - observed azimuth | -60 28 56.000 |
| sum = v | <u>0.305</u> |

which gives the same result.

The geodetic values given above are for the Clarke 1866 ellipsoid ($a = 6378206.4$ m, $1/f = 294.9786982$). The corresponding rectangular coordinates are

| | <u>Standpoint</u> | <u>Forepoint</u> |
|---|-------------------|------------------|
| X | 5528801.2203 m | 5511014.4233 m |
| Y | 0.0000 | 68936.5522 |
| Z | 3170450.6373 | 3205257.0771 |

Let us now translate the coordinates by

$$dX = -10 \text{ m} \qquad dY = 150 \text{ m} \qquad dZ = 170 \text{ m}$$

and choose the WGS 72 ellipsoid ($a = 6378135$ m, $1/f = 298.26$) for expressing geodetic coordinates. This gives

| | <u>Standpoint</u> | <u>Forepoint</u> |
|-----------|-------------------|------------------|
| ϕ | 29°59'58.25797" | 30°20'58.16856" |
| λ | 0 00 05.59611 | 0 43 05.61796 |
| h | 588.3624 m | 3089.6486 m |

If we now apply eqs. (3.1) to (3.9) with the new data and with true ϕ' and λ' values, the true astronomic azimuth will be computed as 60°28'56.305", precisely the same value as before, because ΔX , ΔY , ΔZ have not changed and because the space inverse formula does not recognize any ellipsoid. On the other hand, if the assumed ϕ' and λ' are chosen to be the same as geodetic values, a different result will be obtained for the assumed azimuth, but the final result will be the same as before. We now have

| | | |
|--------------------|---------------|----------------------------|
| computed azimuth | 60°28'56.448" | |
| - observed azimuth | -60 28 56.000 | |
| + $a_7 d\phi'$ | 0.149 | ($d\phi' = 6.742''$) |
| + $a_8 d\lambda'$ | -0.292 | ($d\lambda' = -0.596''$) |
| sum = v | <u>0.305</u> | |

In the classical treatment of this example the required corrections to the astronomic azimuth to reduce it to the ellipsoid (less geodesic and skew normals corrections) are

| | |
|--|---------------|
| - $\sin \phi_1 (\lambda'_1 - \lambda_1)$ | 0.298" |
| - $\sin A_1 \tan V_1 (\phi'_1 - \phi_1)$ | -0.149 |
| + $\cos \phi_1 \cos A_1 \tan V_1 (\lambda'_1 - \lambda_1)$ | <u>-0.006</u> |
| sum | 0.143 |

This is the same as $-(0.149 - 0.292) = 0.143$ obtained previously, demonstrating the equivalency of the traditional and three-dimensional computations.

It should be stressed that in three-dimensional geodesy the same results will be obtained without using the values ϕ , λ , h anywhere in the computations, since the ellipsoid does not enter into the picture at all, except when it is introduced deliberately either for operational convenience or for admitting the geoid into the computational system.

8. WEIGHTS

The weight of an observation is the reciprocal of the square of its standard deviation. An input card prescribes the standard deviation values for the various classes of observations, but this can be overridden by the values given on individual observation cards.

The standard deviations for directions, azimuths, distances, and vertical angles may be specified in two components, for example, as $\pm 1 \text{ mm} \pm 0.8''$ for a direction or as $\pm 5 \text{ mm} \pm 1 \text{ ppm}$ for a distance. The two components are converted to the same units and combined vectorially to give one value.

An unknown is effectively fixed by assigning a very small standard deviation to its observation equation. The default value for fixing station coordinates is 1 mm in each component. The default value for astronomic latitudes and longitudes is 0.01".

9. ERROR ANALYSIS

The adjustment package returns a full variance-covariance matrix of adjusted unknowns. Its diagonal terms q_{xx} are used to compute the standard errors of adjusted unknowns, except for orientation unknowns. The full matrix is used to evaluate standard errors of functions of the unknowns and correlation coefficients by the usual formulas.

The standard error of an unknown is given by

$$\sigma_x = \sigma_0 \sqrt{q_{xx}}, \quad (9.1)$$

where

$$\sigma_0^2 = \frac{[pvv]}{n - u} \quad (9.2)$$

is the variance of unit weight, n and u denote the number of observations and the number of unknowns respectively, p is the weight, and v is the residual.

For the purpose of error analysis over a selected line, a subroutine first builds an 8×8 matrix Q_x , composed of variances of the unknowns

$$dx_1, dy_1, dh_1, dx_2, dy_2, dh_2, d\phi'_1, d\lambda'_1$$

and of the covariances between them.

Three additional matrices are built:

$$F^T = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & 0 & 0 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \end{bmatrix} \quad (9.3)$$

$$D = \begin{bmatrix} -Q_1 & P_1 & 0 \\ P_1/S & Q_1/S & T_1/S \\ -P_1T_1/R_1 & -Q_1T_1/R_1 & R_1 \end{bmatrix}$$

$$= \begin{bmatrix} -S \sin A_1 \cos V_1 & S \cos A_1 \cos V_1 & 0 \\ \cos A_1 \cos V_1 & \sin A_1 \cos V_1 & \sin V_1 \\ -S \cos A_1 \sin V_1 & -S \sin A_1 \sin V_1 & S \cos V_1 \end{bmatrix} \quad (9.4)$$

and the matrix R_0 , as given by (3.22).

The variance-covariance matrix of adjusted astronomic azimuth, spatial distance, and astronomic vertical angle is given by

$$M_{(A,S,V)} = F^T Q_x F. \quad (9.5)$$

Let G have the same elements as F , except for a_7 , a_8 , c_7 , and c_8 which are set to zero. Then the variance-covariance matrix of coordinate differences P , Q , T in the horizon system

is given by

$$M_{\Delta X} = D^T G^T Q_x G D. \quad (9.6)$$

The matrix $M_{\Delta X}$ is rotated to the equatorial system to give the variance-covariance matrix of coordinate differences $\Delta X, \Delta Y, \Delta Z$:

$$M_{\Delta X} = R^T M_{\Delta X} R. \quad (9.7)$$

Coefficients of correlation for functions of adjusted unknowns are computed from $M_{(A,S,V)}$ and $M_{\Delta X}$. The value of each correlation coefficient is given by

$$r_{ij} = \frac{q_{ij}}{\sqrt{q_{ii}q_{jj}}}. \quad (9.8)$$

In the solution with invariable astronomic coordinates the Q_x matrix is extended to an 8x8 matrix. Its seventh and eighth diagonal elements are then the a priori variances of astronomic latitude and longitude of the standpoint, both divided by σ_0^2 , and off-diagonal elements in the last two rows and columns are all zeros.

10. DIAGNOSTIC MESSAGES

Diagnostic messages have been incorporated in the program for several situations when something is wrong with the input. When a mistake is found, the program may continue or terminate the execution, depending on the severity of the error.

The stations are identified within the program by numbers that are assigned by the user in addition to station names. After the stations have been read in, the program searches for duplicate station numbers and prints a message if one is found. A duplicate station number is a fatal error; however, this does not stop the program until all observations have been read and inspected.

Each observation is assigned a unique line number by the program. If an observation is from or to a station that is not included in the station list, its line number is printed as a negative value and the observation is ignored by the program. Execution is continued.

A message is given for each observation equation that gives the absolute value of the constant term as larger than 70 times the assigned standard deviation. Very often this is due only to imprecision of preliminary coordinates and does not hurt the solution, but sometimes it may point to a source of trouble in

the input. Only 50 such messages are allowed to be printed, and if after that an observation is found with the constant term 300 times larger than its standard deviation, the program stops.

A solution is not attempted if the number of unknowns exceeds the number of observations. The program prints a message and stops.

The program identifies a singular matrix and the unknowns which contributed to the singularity.

Satisfactory convergence is assumed to have been attained if the total shift of each station from the previous to the current position is within 0.01 m. The program imposes a limit of three iterations, but this can be changed downward or upward to nine iterations. If the adjustment has not converged, the program prints a message and continues with the rest of the solution and the output.

Other diagnostic messages of lesser interest have also been included.

11. MISCELLANEOUS COMPUTATIONS

An option is included in the program to transform adjusted coordinates to a different reference system by specifying three translation components, three rotation angles, and a scale correction. The translation components are ΔX , ΔY , and ΔZ , new minus old values. The rotation angles about the respective axes are ϵ , ψ , and ω , counterclockwise when viewed from the positive axis toward the origin. The scale difference Δc means that the coordinates in the old system should be multiplied by $(1 + \Delta c)$ to give the corresponding coordinates in the new system. The new coordinates X' , Y' , Z' are given by

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 0 & \omega & -\psi \\ -\omega & 0 & \epsilon \\ \psi & -\epsilon & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \Delta c \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}. \quad (11.1)$$

Geodesic azimuths and distances are computed over selected lines after the adjustment by a modification of the Helmert iterative method (Vincenty 1975, 1976).

If the parameters of another ellipsoid are furnished in the input, the program recomputes the adjusted coordinates in terms of the second ellipsoid by eqs. (3.16) to (3.21).

REFERENCES

- Bartelme, N. and Meissl, P., 1975: Ein einfaches, rasches und numerisch stabiles Verfahren zur Bestimmung des kürzesten Abstandes eines Punktes von einem sphäroidischen Rotationsellipsoid. Allgemeine Vermessungs-Nachrichten, Heft 12, 436-439.
- Bopp, H. and Krauss, H., 1976: Der kürzeste Abstand eines Punktes von einem Rotationsellipsoid. Allgemeine Vermessungs-Nachrichten, Heft 10, 348-351.
- Bowring, B. R., 1976: Transformation from spatial to geographical coordinates. Survey Review, No. 181, 323-327.
- Carter, W. E. and Vincenty, T., 1978: Survey of the McDonald Observatory radial line scheme by relative lateration techniques. NOAA Technical Report NOS 74 NGS 9, 33 p.
- Heiskanen, W. A. and Moritz, H., 1967: Physical Geodesy. Freeman and Co., San Francisco and London, 364 p.
- Mitchell, M. M. (The Ohio State University, Columbus) 1963: Fundamentals of spatial coordinates in terrestrial geodesy. M. S. thesis, 81 p.
- Rapp, R. H., 1975: Geometric Geodesy Notes, Vol. II. The Ohio State University, Columbus, 135 p.
- Sikonia, W. G., 1977: Three-dimensional geodetic survey adjustment. U. S. Geological Survey Computer Contribution, 189 pp. Available from U. S. Dept. of Commerce, National Technical Information Service, Springfield, VA 22151, as accession no. PB-278660.
- Vincenty, T., 1975: Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations. Survey Review, No. 175, 88-93.
- Vincenty, T., 1976: Solutions of geodesics. Survey Review, No. 180, 294.
- Vincenty, T., 1978: The use of the relative lateration method for reducing errors in length measurements. Survey Review, No. 189, 295-302.
- Vincenty, T. and Bowring, B. R., 1978: Application of three-dimensional geodesy to adjustments of horizontal networks. NOAA Technical Memorandum NOS NGS-13, 7 p.
- Wolf, H., 1963: Die Grundgleichungen der dreidimensionalen Geodäsie in elementarer Darstellung. Zeitschrift für Vermessungswesen, Heft 6, 225-233.

(Continued from inside front cover)

- NOS NGS-6 Determination of North American Datum 1983 coordinates of map corners. T. Vincenty, October 1976, 8 pp (PB262442). Predictions of changes in coordinates of map corners are detailed.
- NOS NGS-7 Recent elevation change in Southern California. S.R. Holdahl, February 1977, 19 pp (PB265-940). Velocities of elevation change were determined from Southern Calif. leveling data for 1906-62 and 1959-76 epochs.
- NOS NGS-8 Establishment of calibration base lines. Joseph F. Dracup, Charles J. Fronczek, and Raymond W. Tomlinson, August 1977, 22 pp (PB277130). Specifications are given for establishing calibration base lines.
- NOS NGS-9 National Geodetic Survey publications on surveying and geodesy 1976. September 1977, 17 pp (PB275181). Compilation lists publications authored by NGS staff in 1976, source availability for out-of-print Coast and Geodetic Survey publications, and subscription information on the Geodetic Control Data Automatic Mailing List.
- NOS NGS-10 Use of calibration base lines. Charles J. Fronczek, December 1977, 38 pp (PB279574). Detailed explanation allows the user to evaluate electromagnetic distance measuring instruments.
- NOS NGS-11 Applicability of array algebra. Richard A. Snay, February 1978, 22 pp (PB281196). Conditions required for the transformation from matrix equations into computationally more efficient array equations are considered.
- NOS NGS-12 The TRAV-10 horizontal network adjustment program. Charles R. Schwarz, April 1978, 52 pp (PB283087). The design, objectives, and specifications of the horizontal control adjustment program are presented.
- NOS NGS-13 Application of three-dimensional geodesy to adjustments of horizontal networks. T. Vincenty and B. R. Bowring, June 1978, 7 pp (PB286672). A method is given for adjusting measurements in three-dimensional space without reducing them to any computational surface.
- NOS NGS-14 Solvability analysis of geodetic networks using logical geometry. Richard A. Snay, October 1978, 29 pp (PB291286). No algorithm based solely on logical geometry has been found that can unerringly distinguish between solvable and unsolvable horizontal networks. For leveling networks such an algorithm is well known.
- NOS NGS-15 Goldstone validation survey - phase I. William E. Carter and James E. Pettey, November 1978, 44 pp (PB292310). Results are given for a space system validation study conducted at the Goldstone, Calif., Deep Space Communication Complex.
- NOS NGS-16 Determination of North American Datum 1983 coordinates of map corners (Second Prediction). T. Vincenty, April 1979, 6 pp. New predictions of changes in coordinates of map corners are given.

NOAA Technical Reports, NOS/NGS subseries

- NOS 65 NGS 1 The statistics of residuals and the detection of outliers. Allen J. Pope, May 1976, 133 pp (PB258428). A criterion for rejection of bad geodetic data is derived on the basis of residuals from a simultaneous least-squares adjustment. Subroutine TAURE is included.
- NOS 66 NGS 2 Effect of Geceiver observations upon the classical triangulation network. R. E. Moose and S. W. Henriksen, June 1976, 65 pp (PB260921). The use of Geceiver observations is investigated as a means of improving triangulation network adjustment results.
- NOS 67 NGS 3 Algorithms for computing the geopotential using a simple-layer density model. Foster Morrison, March 1977, 41 pp (PB266967). Several algorithms are developed for computing with high accuracy the gravitational attraction of a simple-density layer at arbitrary altitudes. Computer program is included.

(Continued on inside back cover)

(Continued)

- NOS 70 NGS 5 Selenocentric geodetic reference system. Frederick J. Doyle, Atef A. Ellassal, and James R. Lucas, February 1977, 53 pp (PB266046). Reference system was established by simultaneous adjustment of 1,233 metric-camera photographs of the lunar surface from which 2,662 terrain points were positioned.
- NOS 71 NGS 6 Application of digital filtering to satellite geodesy. C. C. Goad, May 1977, 73 pp (PB-270192). Variations in the orbit of GEOS-3 were analyzed for M_2 tidal harmonic coefficient values which perturb the orbits of artificial satellites and the Moon.
- NOS 72 NGS 7 Systems for the determination of polar motion. Soren W. Henriksen, May 1977, 55 pp (PB274698). Methods for determining polar motion are described and their advantages and disadvantages compared.
- NOS 73 NGS 8 Control leveling. Charles T. Whalen, May 1978, 23 pp (GPO# 003-017-00422-8) (PB286838). The history of the National network of geodetic control, from its origin in 1878, is presented in addition to the latest observational and computational procedures.
- NOS 74 NGS 9 Survey of the McDonald Observatory radial line scheme by relative lateration techniques. William E. Carter and T. Vincenty, June 1978, 33 pp (PB287427). Results of experimental application of the "ratio method" of electromagnetic distance measurements are given for high resolution crustal deformation studies in the vicinity of the McDonald Lunar Laser Ranging and Harvard Radio Astronomy Stations.
- NOS 75 NGS 10 An algorithm to compute the eigenvectors of a symmetric matrix. E. Schmid, August 1978, 5 pp (PB287923). Method describes computations for eigenvalues and eigenvectors of a symmetric matrix.
- NOS 76 NGS 11 The application of multiquadric equations and point mass anomaly models to crustal movement studies. Rolland L. Hardy, November 1978, 63 pp (PB293544). Multiquadric equations both harmonic and non-harmonic, are suitable as geometric prediction functions for surface deformation and have potentiality for usage in analysis of subsurface mass redistribution associated with crustal movements.

NOAA Manuals, NOS/NGS subseries

- NOS NGS 1 Geodetic bench marks. Lt. Richard P. Floyd, September 1978, 56 pp. Reference guide provides specifications for highly stable bench marks, including chapters on installation procedures, vertical instability, and site selection considerations.

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Survey
National Geodetic Survey, C13x4
Rockville, Maryland 20852

OFFICIAL BUSINESS

POSTAGE AND FEES PAID
U.S. DEPARTMENT OF COMMERCE
COM-210
THIRD CLASS MAIL



NOAA--S/T 79-219