DATUM DEFINITION STUDY FOR THE

NORTH AMERICAN VERTICAL DATUM OF 1988

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David B. Zilkoski
Emery I. Balazs
Janice M. Bengston
Vertical Network Branch
National Geodetic Survey
Coast & Geodetic Survey
National Ocean Service, NOAA
Rockville, MD 20852

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EXECUTIVE SUMMARY

This report describes the investigation of several datum definition scenarios associated with the new adjustment of the North American Vertical Datum of 1988 (NAVD 88). The report begins with a brief history and development of the vertical control portion of the National Geodetic Reference System (NGRS). Current progress of other aspects of the new adjustment of NAVD 88 by the National Geodetic Survey (NGS) is also discussed.

The new adjustment of NAVD 88 received approval and funding in fiscal year 1978. An important feature of the program involves releveling 81,500 km of first-order leveling lines to strengthen the network. Other major tasks include block validation (final quality assurance) and Helmert blocking. Data outliers are detected and removed during these tasks.

Since each region of the country is unique, the impact of NAVD 88 will be slightly different for each area. To assist in identifying and documenting the impact of the new datum, NGS compiled a primary vertical control network for the conterminous United States using the latest leveling data available. Analyses of this network were helpful in determining the effects of various datum constraints, magnitudes of height changes from the present National Geodetic Vertical Datum of 1929 (NGVD 29), influences of systematic errors, deficiencies in network design, and additional releveling requirements. The results of these analyses are discussed.

Adjustments imposing various constraints were performed. A comparison of the different datum constraints and their effects on heights at junction bench marks, differences between NGVD 29 and preliminary NAVD 88 values at junction bench marks, and differences between local mean sea level (LMSL) (1960-78 epoch) and preliminary NAVD 88 values at primary tidal bench marks were investigated.

In most "stable" areas, relative height changes between adjacent bench marks should be only a few millimeters. Some absolute height values will change much more. Preliminary results indicate that many bench mark height values will change by 50 to 75 cm, with some changing as much as 150 cm. The differences between NAVD 88 and NGVD 29 are caused by many factors. It is difficult to separate the overall change in bench mark height into its individual components, such as the effects of systematic errors, crustal motion, and datum distortions.

Comparisons of adjusted heights indicate that most corrections for systematic errors do not significantly change adjusted heights in a continental sense. However, in some regions the corrections do have a large local effect. Investigations of general adjustment results for NGVD 29 indicate that large adjustment corrections (residuals) were distributed in some areas of the country. For example, the accumulated 1929 adjustment correction for an east-west leveling route running 3,000 km from Crookston, MN, to Seattle, WA, was 89 cm.

In areas of vertical crustal motion, the amount of relative height changes will also depend on the magnitude of the actual physical movement of the bench marks. In many stable areas a single bias factor, describing the difference between NGVD 29 and NAVD 88, can be estimated and used for most mapping applications.

Comparisons of the primary network adjusted heights with values obtained in other ways show good agreement. Adjusted heights from this study, when compared to uncorrected, observed Canadian geodetic leveling agree to less than 50 cm from coast to coast, with a possibility that this difference will decrease when all data are included to form a network and after corrections are applied to account for known systematic effects. In another comparison, substituting the estimates of orthometric heights from this study for published NGVD 29 values significantly reduced an apparent 2-meter bias between modeled geoid heights and satellite-derived geoid heights (ellipsoid heights minus orthometric heights) at laser stations located in the western United States.

ABSTRACT

The new adjustment of the North American Datum of 1988 (NAVD 88) received approval and funding in fiscal year 1978. In most "stable" areas, relative height changes between adjacent bench marks should be less than 1 cm. Analyses indicate that some absolute height values will change much more. These differences are due to many factors, such as large distribution corrections (residuals) from past adjustments, better estimated of corrections applied to account for systematic errors, and geopotential differences using actual gravity values instead of normal orthometric height differences. Preliminary results indicate that many bench mark height values will

change form 50 to 75 cm, with some changing as much as 150 cm. The differences between NAVD 88 and the present National Geodetic Vertical Datum of 1929 (NGVD 29) are caused by many factors. An investigation of NGVD 29 general adjustment results indicates that rather large adjustment corrections (residuals) were distributed in some areas of the country. For example, the accumulated 1929 adjustment correction along a 3,000 km east-west leveling route form Crookston, Minnesota, to Seattle, Washington, was 89 cm. It is difficult to separate the overall change in bench mark height into its individual components such as the effects of systematic error, crustal movement, and datum distortion.

BACKGROUND

The first leveling route in the United States considered to be of geodetic quality was established in 1856-57 under the direction of G. B. Vose of the U.S. Coast Survey (predecessor of the Coast and Geodetic Survey, now the National Ocean Service). The leveling survey was undertaken to support current and tide studies in the New York Bay and Hudson River areas. The first leveling line officially designated as "geodesic leveling" by the Coast and Geodetic Survey followed an arc of triangulation along the 39th parallel. This 1887 survey began at bench mark A in Hagerstown, MD.

By 1900, the vertical control network had grown to 21,095 km of geodetic leveling. Data included work performed by the Coast and Geodetic Survey, U.S. Army Corps of Engineers, U.S. Geological Survey, and the Pennsylvania Railroad. A reference surface was determined in 1900 by holding elevations referenced to local mean sea level fixed at five tide stations. Data from two other tide stations indirectly influenced the determination of the reference surface. Subsequent readjustments of the leveling network were performed by the Coast and Geodetic Survey in 1903, 1907, and 1912 (Berry 1976).

The next general adjustment of the vertical control network was accomplished in 1929. By then the international nature of geodetic networks was well understood and Canada provided data for its first-order vertical network to combine with the U.S. net. The two networks were connected at 24 locations through vertical control points (bench marks) from Maine/New Brunswick to Washington/British Columbia. Although Canada did not adopt the "Sea Level Datum of 1929" determined by the United States, Canadian-U.S. cooperation in the general readjustment greatly strengthened the 1929 network. Table 1 lists the kilometers of leveling involved in the readjustments and the number of tide stations used to establish the datums.

Table 1.--Amount of leveling and number of tide stations involved in previous readjustments

Year of adjustment	Kilometers of leveling	Number of tide stations
1900	21,095	5
1903	31,789	8
1907	38,359	8
1912	46,462	9
1929	75,159 (U.S.)	21 (U.S.)
	31,565 (Canada)	5 (Canada)

NEW ADJUSTMENT OF THE NORTH AMERICAN VERTICAL DATUM OF 1988

Approximately 625,000 km of leveling have been added to NGRS since the 1929 adjustment. In the intervening years, numerous discussions were held to determine the proper time for the inevitable new general adjustment. In the early 1970s, NGS conducted an extensive inventory of the vertical control network. The search identified thousands of bench marks that had been destroyed, due primarily to post-World War II highway construction. Many existing bench marks were affected by crustal motion associated with earthquake activity, post-glacial rebound (uplift), or subsidence resulting from withdrawal of underground liquids. Other problems (distortions in the network) were caused by forcing the 625,000 km of leveling to fit previously determined NGVD 29 height values. Some observed changes, amounting to as much as 9 m, are discussed in previous reports (Zilkoski 1986, Zilkoski and Young 1985).

In Fiscal Year 1977, NGS prepared a budget initiative to finance the readjustment. The revised plan, which was later approved and funded by the Department of Commerce, identified the project as the North American Vertical Datum of 1988 (NAVD 88). Activities formally began in October 1977. The project, scheduled for completion in 1991, has dominated NGS' Vertical Network Branch (VNB) activities since its beginning. Details of major NAVD 88 tasks are described in previous reports (Zilkoski 1986, Zilkoski and Young 1985).

During the past year, most VNB personnel were involved with the block validation effort. The block validation process combined and analyzed all observed elevation differences in a predefined area (Bengston 1986). During the analysis, a first-order primary network consisting of the latest data was selected and analyzed, and the results documented. Appropriate remaining leveling data were then incorporated into the first-order network. Data outliers were detected and removed during this task. Data involving 585,000 bench marks were processed. After block validation, which was completed in October 1989, the next major task is Helmert blocking.

Helmert blocking consists of partitioning 1.3 million unknowns (approximately 600,000 permanently monumented bench marks and 700,000 "temporary" bench marks) and associated observations into manageable blocks. A least squares adjustment is then performed on the entire data set. Helmert blocking began, in a production mode, in October 1989, with the final adjustment targeted for completion in April 1991.

An important feature of the NAVD 88 program is the releveling of much of the first-order NGS vertical control network in the United States. The dynamic nature of the network requires a framework of newly observed height differences to obtain realistic, contemporary height values from the readjustment. To accomplish this, NGS identified 81,500 km for releveling.

Replacement of disturbed and destroyed monuments precedes the actual releveling. This effort also includes the establishment of stable "deep-rod" bench marks, which will provide reference points for future "traditional" and "satellite" leveling systems. Field leveling is being accomplished to Federal Geodetic Control Committee (FGCC) first-order, class II specifications, using the "double-simultaneous" method (Whalen and Balazs 1976).

DATUM DEFINITION IMPLICATIONS

For the NGVD 29 general adjustment, heights of 26 tidal bench marks referenced to local mean sea level were rigidly constrained to define a reference surface (datum) based on a value of 0.0 m for each local mean sea level. A change in philosophy for NAVD 88 could have a major impact on mapping agencies, e.g., the U.S. Geological Survey (USGS) and the Federal Emergency Management Agency (FEMA) (Miller 1985, Southard 1985, Zilkoski 1986).

The impact of NAVD 88 will be slightly different in each area of the country. In "stable" areas, relative height changes between adjacent bench marks should be only a few millimeters. The absolute heights of some bench marks will change much more. Because datum definition could change absolute heights by a significant amount, it deserves serious consideration by the agencies involved. In many stable areas a single bias factor, describing the difference between NGVD 29 and NAVD 88, could be estimated by comparing common bench mark heights in both systems. Even with this factor, NAVD 88 will still have a major impact on mapping agencies.

FEMA (Miller 1985) states that if the relative height changes between bench marks remain fairly constant over a given geographic area, the impact to the National Flood Insurance Program (NFIP) will be minimal. As long as the relative hydrographic conditions shown on the FEMA maps remain correct, FEMA has to ensure only that consistent data are used when comparing flood elevation to structure elevations (Knoderer 1990).

Therefore, a bias shift in heights resulting from a change in datum definition for NAVD 88 should not have a major impact on NFIP. FEMA requests assistance in educating the user. Currently, more than 17,500 communities participate in NFIP through 150,000 insurance agents who represent 2 million flood insurance policy holders. Miller suggests that technical reports be prepared by NGS for engineers and surveyors, and non-technical reports for others.

The U.S. Geological Survey (Southard 1985, Chapman 1990) stresses that datum definition is critical to the National Mapping Program. USGS produces 60,000 different map products. The 7.5-minute series will be the one most affected by a datum change. USGS has approximately 100 spot elevations on each 7.5-minute quad, as well as various contour intervals. One-tenth of a contour interval can be handled by adding a statement in the margin for datum shift between NGVD 29 and NAVD 88. In flat terrain, a datum shift exceeding 1 foot (30 cm) will present problems; in mountainous regions, shifts in excess of 8 feet (244 cm) will create problems. The total conversion of all USGS maps could cost as much as $45 million.

Thousands of other data bases and maps are based on NGVD 29 heights, which may have to be updated to be consistent with NAVD 88 height values. All bench mark values published by NGS will have new NAVD 88 heights. Updating these values will not be a difficult task, but it probably will be time-consuming and expensive. A larger problem will be the estimation of heights for bench marks that are not part of the National Geodetic Reference System. The only rigorous method of incorporating these bench marks into NAVD 88 is to process and analyze the original observations, and then fit them to NAVD 88. This could also be time-consuming and expensive. Once again, a factor describing the approximate separation between NGVD 29 and NAVD 88 could be estimated. Depending on users' requirements, this factor may be sufficient. The Vertical Network Branch is prepared to assist users in evaluating their situation and developing a plan to convert their heights from NGVD 29 to NAVD 88.

In areas of crustal motion, relative height changes will also depend on the magnitude of actual ground movement. NGS is developing crustal motion models and publishing estimates of bench mark velocities wherever enough data exist, e.g., portions of California.

DATUM DEFINITION TASKS

In the past, heights of tidal bench marks referenced to local mean sea level (LMSL) at selected tidal stations were rigidly constrained to define a reference surface for orthometric heights; i.e., a "mean sea level" reference surface was determined for NGVD 29 by assuming the heights of LMSL at 26 tidal stations to be zero. Theoretical geodesists would prefer the new 1988 datum to be based on an equipotential surface or a surface that closely approximates an equipotential surface. It is well known that local mean sea level determined by tidal data at different sites does not lie on the same equipotential surface. The difference between measured local mean sea levels and a global equipotential surface, coincident with LMSL at one point, is due to the effects of sea surface topography.

Heights of tidal bench marks referenced to LMSL or height differences between tidal bench marks should be incorporated into NAVD 88. However, there are still some unanswered questions. How do we best incorporate and assign weights to tidal height observations? Can we properly estimate the effects of SST at tidal stations? What is the best method to use satellite Global Positioning System information to reduce datum distortions? The answer to the first question depends largely on the answer to the second question.

The datum definition task is one of the last NAVD 88 tasks to be completed. However, there are many factors which need to be considered before a decision can be made. The following questions must be answered before the new datum for NAVD 88 can be defined:

1. Should LMSL values at all tidal stations be held fixed at zero to minimize impact on the mapping community? What will be the effect upon others if LMSL values are held at zero?

2. Are differences between LMSL values and observed geodetic leveling height differences due to the effects of sea surface topography or errors in leveling data?

3. What is the real impact on the mapping community? Can one LMSL value be held fixed to define the datum, and then a vertical block shift performed to minimize height discrepancies between local mean sea levels and NAVD 88 in such a manner that the impact to users is minimal? (That is, the smallest height discrepancies occur in low-lying regions of the country.)

4. How accurately can the effects of sea surface topography be estimated? Is the benefit/cost ratio of estimating these effects too low, considering the primary use of NAVD 88 is for engineering purposes?

5. Should NGS distort the observations and define an engineering datum, which would meet about 95 percent of users' needs, and create separate, task-specific scientific "datums" upon request?

6. What are the legal implications of a datum change? How many state laws, zoning regulations, and other statutes are written based on NGVD 29?

ANALYSES OF NAVD 88 PRIMARY VERTICAL CONTROL NETWORK

The Vertical Network Branch has undertaken a special study to compile a primary vertical control network using the latest data available. (See figure 1.) Analyses of this network were helpful in determining the effects of various datum constraints and the magnitudes of height changes from NGVD 29.

Most of the data in the study were observed between 1965 and 1986, but some older data, obtained in the 1940s and 1950s, were included to reduce the size of some loops. These data are located mostly in the Great Plains and the Pacific coast. Inclusion of these data did not affect the analysis in a continental sense, but probably did influence estimates of heights locally. The local effect, however, should be small because many of these older leveling lines were rejected during the analyses. (See figure 2.) This will be discussed in greater detail later. During fiscal years 1987 through 1990, several thousand kilometers of leveling data were obtained in the Pacific coast region of the United States. As noted above these data were not available at the time this study was performed. However, subsequent studies, re-analyzing adjusted heights using these latest data in the Pacific coast region were performed and some results are included in appendix A of this report. The analyses performed agreed with the results documented in this report.

This primary network of 200 loops contains 909 junction bench marks. Each loop is composed of links based on the latest leveling data that connect the junctions of loops. The network connects to 57 primary tidal stations which are part of the National Primary Tidal Network and 55 water-level stations along the Great Lakes. In addition, 28 connections were made to the Canadian vertical control network and 13 to the Mexican vertical control network.

The leveling observations were corrected for rod scale, rod temperature, level collimation, astronomic, refraction, and magnetic effects (Balazs and Young 1982, Holdahl et al. 1986). All geopotential differences were generated and validated, using gravity values derived from a 4-kilometer gridded Bouguer anomaly data set provided by the Society of Exploration Geophysicists. Loop misclosures were computed and checked against allowable tolerances. Geopotential differences were used as observations in the least squares adjustment, geopotential numbers were solved for as unknowns, and orthometric heights were computed using the well known Helmert height reduction (Helmert 1890): $H = C/(g + 0.0424H)$, where C is the estimated geopotential number in gpu, g is the gravity value at the bench mark in gals, and H is the orthometric height in kilometers. The weight of an observation was calculated using the formula 1/(variance of the observation), where the variance of the observation is equal to (a priori standard error ^{2} X kilometers of leveling)/(number of runnings). The a priori standard errors for all orders and classes of leveling defined by FGCC are as follow:

First-order, class 0	= 0.7 mm
First-order, class I	= 1.1 mm
First-order, class II	= 1.4 mm
Second-order, class I	= 2.1 mm
Second-order, class II	= 2.8 mm
Second-order, class 0	= 3.0 mm
Third-order	= 4.2 mm

Heights of bench marks were computed using a least squares adjustment. Data outliers were detected and removed during this analysis. Thirty-six links were rejected because of larger than expected misclosures. Most of these links involved connections between "old" and "new" leveling data. After the data outliers were removed, the primary network consisted of 179 loops and 896 junction bench marks. (See figure 2.) Tables 2-3 and figures 3-4 provide some general statistics obtained during these analyses.

The standard error of unit weight is not very useful at this time because the relative weighting scheme is not known very well. Older first-order leveling data were given the same weight as newer first-order data. A statistical analysis of NAVD 88 results after the general adjustment will provide more reliable values. Studies estimating appropriate a priori standard errors of leveling data will be performed prior to the general adjustment.

Table 2.--Summary of statistics from minimum-constraint least squares adjustments

	No. of BMS	No. of obs.	No. of obs. rejected	Std. error of unit weight	Degrees of freedom
All data included	909	1,116	0	2.5	208
Outliers remove	896	1,080	36	1.9	185

Adjustments imposing various datum definition scenarios were performed. A comparison of the different datum constraints and their effects on heights at the junction bench marks, differences between NGVD 29 and preliminary NAVD 88 values at junction bench marks, and differences between LMSL (1960-78 epoch) and preliminary

NAVD 88 values at the primary tidal bench marks are discussed in this report.

Results of Loop Analyses

After all geopotential differences were generated and validated, loop misclosures were computed and checked against allowable tolerances. Table 3 and figures 3-4 give some general statistics about loop closures, tables 4-5 and figure 5 provide additional detail. Figure 5 shows the location of loops by identification members as listed in tables 4-5. Loops which were outside allowable limits are highlighted in figure 5. Figure 2 shows the links which were rejected to reduce the effects of data outliers on the adjusted heights. These links are being investigated to determine why they disagree with surrounding data. Most loop misclosures which are outside their allowable tolerance involved connections between "old" and "new" data, e.g., loops 188, 192, 194, 245, 252A, and 257A. (See figure 5.) Some of these leveling lines are scheduled to be releveled as part of this project. When data become available, special studies will be performed to investigate the Pacific coast and Great Plains regions using the new data.

Table 3.--Summary of statistics from loop misclosure analysis

	No. of loops by sign			No. of loops outside allowable limit			No. of loops within allowable limit		
	Neg.	Pos.	Total	Neg.	Pos.	Total	Neg.	Pos.	Total
All data included	107	93	200	16	12	28	91	81	172
Outliers removed	97	82	179	2	2	4	95	80	175

For each loop, table 4 gives the distance of the loop in kilometers, the allowable misclosure based on FGCC specifications (FGCC 1984), and the misclosures of the loops (computed in a clockwise direction) both with and without corrections applied for systematic errors. A few items should be pointed out in table 4. First, 28 loop misclosures were outside their allowable limits when all corrections were applied, 29 loop misclosures were outside their allowable limits when all corrections except the refraction correction were applied, and 23 loop misclosures out of a total of 96 that were influenced by magnetic error were outside their allowable limits when all corrections except the magnetic correction

Table 4 Kgal-mm

LOOP CLOSURES (MM.)

LOOP	DIST. (KM.)	ALLOW (MM.)	ALL CORR.	W/O MAG.	W/O REF.	W/O ROD.	W/O LEVEL.	W/O TEMP.	W/O ASTRO.
1	763.22	132.36	*** 190.70	*** 335.91	*** 192.01	*** 189.92	*** 191.89	*** 190.64	*** 193.38
2	658.32	124.76	-101.21	-63.15	-100.71	-100.95	-101.71	-99.77	-101.92
3	860.06	142.76	19.51	100.77	5.11	24.39	18.32	17.94	16.93
4	1305.18	166.48	-15.36	-15.38	-4.32	-18.58	-15.58	-21.84	-21.23
6	1517.21	182.97	61.66	61.66	58.64	52.89	64.01	63.79	61.30
7	1002.49	158.31	-113.47	-113.47	-112.94	-104.28	-113.23	-116.21	-111.08
8	891.99	144.18	-20.16	-18.65	-18.68	-26.33	-20.74	-17.13	-21.64
9	456.46	102.83	34.40	48.76	36.49	39.87	35.93	34.88	33.63
10	588.73	113.98	-23.60	-24.77	-12.06	-16.10	-24.79	-22.71	-25.72
11	760.36	122.02	-13.86	46.67	-12.31	-14.15	-12.33	-12.53	-10.32
12	531.11	99.13	33.77	-11.36	29.61	33.87	33.89	33.61	33.33
13	448.99	96.55	-52.51	-50.71	-55.13	-60.46	-52.62	-53.32	-51.30
14	523.72	114.42	73.38	31.04	71.37	73.65	73.06	73.26	73.35
15	389.63	94.63	53.59	*** 115.24	52.54	55.31	54.30	49.75	49.98
16	527.39	111.53	75.57	34.91	72.66	71.46	76.03	79.93	74.93
17	876.48	144.53	15.28	34.83	13.14	15.85	15.22	14.39	17.13
18	988.00	140.77	-73.56	-73.56	-68.81	-64.31	-74.26	-76.81	-78.48
19	1084.20	153.83	-42.85	-42.85	-57.07	-36.34	-43.93	-40.33	-36.38
1A	282.30	80.97	-1.29	-38.89	-1.26	-1.39	-1.53	-1.32	-1.79
1B	592.89	137.94	-4.62	*** -172.33	-4.57	-4.67	-5.16	-4.70	-5.17
20	647.88	125.22	-21.38	-21.38	-15.59	-23.62	-21.13	-15.13	-19.14
21	573.19	104.60	-49.88	-49.88	-38.29	-50.99	-49.35	-50.10	-52.05
22	729.70	123.31	-65.67	-86.88	-76.47	-50.80	-64.13	-65.84	-66.68
23	850.68	137.03	126.25	101.09	*** 151.67	114.12	128.35	132.87	126.07
24	876.96	129.72	3.55	13.53	-11.68	5.41	3.66	2.03	4.20
25	755.74	114.41	-110.39	-28.08	-105.41	-107.12	-109.38	*** -106.22	*** -116.70
26	860.38	123.10	48.18	65.32	48.35	44.85	45.65	43.51	48.75
27	669.57	124.80	-36.25	62.50	-41.95	-40.57	-37.18	-36.74	-35.47
28	694.70	112.91	-36.22	*** -125.59	-33.69	-35.44	-37.99	-38.25	-37.15
29	568.22	110.53	-25.07	-25.07	-23.54	-25.19	-25.93	-22.06	-28.64
30	242.42	77.85	16.08	16.08	15.90	16.41	15.97	15.15	15.98
31	700.92	129.42	-30.97	-30.97	-27.55	-27.75	-31.68	-34.03	-29.49
32	540.07	110.89	40.96	2.26	46.87	40.35	41.25	41.62	38.84
36	526.83	105.90	31.89	21.84	34.44	31.65	32.70	31.60	31.59
37	176.27	61.50	-12.36	-12.36	-12.16	-12.61	-12.26	-13.86	-12.38
38	641.78	121.41	89.29	116.01	94.34	82.18	90.82	91.12	88.92
50	632.15	119.08	-44.25	*** -204.56	-44.49	-47.22	-45.28	-45.11	-48.29
51	555.97	117.89	-37.00	69.53	-34.63	-34.61	-36.57	-38.35	-37.12
52	791.91	140.70	98.76	*** 146.36	96.55	98.26	96.94	102.51	98.49
53	802.77	136.36	-41.77	69.70	-49.25	-42.98	-41.54	-49.87	-43.63
54	1071.42	152.37	32.44	*** 228.71	36.62	27.18	31.49	29.27	28.93
55	703.19	118.52	97.92	97.92	101.56	95.85	97.97	98.50	95.52
56	313.74	70.85	-15.54	-40.44	-16.53	-15.80	-16.77	-15.57	-15.57
57	641.46	110.42	-33.45	-80.77	-33.93	-33.98	-30.79	-33.98	-34.26
58	312.83	78.20	-61.14	10.61	-61.03	-60.04	-61.72	-61.90	-61.08
59	502.52	102.69	-26.94	-26.94	-26.83	-28.15	-25.65	-28.16	-27.08
60	683.11	115.41	-22.13	-22.13	-24.08	-24.52	-20.95	-22.63	-24.12
61	832.18	132.51	7.81	41.60	10.72	9.89	8.26	8.24	13.10
62	697.40	114.06	43.70	9.71	45.56	44.03	43.47	43.74	39.21
63	594.13	97.49	-34.64	*** 113.89	-33.51	-33.77	-33.99	-38.15	-33.99
70	705.06	128.57	-73.23	-73.23	-69.98	-82.18	-72.63	-75.00	-74.71
71	589.35	110.09	*** 128.94	*** 128.94	*** 127.83	*** 122.88	*** 126.52	*** 128.59	*** 126.50
72	982.88	142.26	*** 237.09	*** 237.09	*** 231.90	*** 237.59	*** 238.71	*** 235.85	*** 240.98
73	1638.86	193.83	-19.37	-19.37	-20.77	-22.34	-19.62	-19.03	-18.61

*** - Outside Allowable Tolerance

Table 4 (con't)

LOOP	DIST. (KM.)	ALLOW (MM.)	LOOP CLOSURES (MM.) ALL CORR.	W/O MAG.	W/O REF.	W/O ROD.	W/O LEVEL.	W/O TEMP.	W/O ASTRO.
77	1574.75	180.62	86.49	-20.42	98.20	88.65	87.59	86.18	83.80
79	911.55	133.90	-82.86	-39.06	-69.29	-81.14	-83.81	-81.83	-82.67
80	685.04	130.88	104.21	104.21	109.15	96.87	104.33	107.23	106.61
81	875.94	140.29	-13.25	-104.88	-17.32	-16.84	-12.27	-11.32	-15.38
82	902.65	150.22	-66.37	-66.37	-59.19	-67.83	-66.62	-68.17	-65.68
84	765.16	138.30	50.36	50.36	43.71	50.65	50.35	51.23	50.54
86	719.71	134.13	-103.07	-103.07	-92.93	-104.18	-103.32	-103.02	-104.21
87	720.47	134.20	47.25	47.25	56.77	47.38	47.64	42.84	44.66
88	916.54	151.37	-56.57	-56.57	-29.44	-54.27	-56.82	-44.79	-63.49
90	1390.93	186.47	31.38	30.35	11.65	10.70	31.88	25.41	31.15
93	884.66	148.71	-12.51	-11.49	-5.55	-4.17	-12.59	-9.37	-16.61
94	1074.22	163.87	-85.59	-85.45	-91.28	-68.93	-85.26	-92.76	-67.98
97	918.17	151.50	-23.09	-23.09	3.16	-25.46	-22.49	-10.28	-23.70
98	1041.63	161.38	-11.28	-11.28	26.65	-0.13	-11.76	-14.04	-7.79
101	749.44	130.38	-30.09	-30.09	-58.35	-49.71	-29.55	-23.66	-28.62
102	1076.86	156.81	151.24	151.24	*** 179.19	*** 172.58	151.11	156.93	147.92
103	817.75	134.58	-58.71	*** -140.95	-56.61	-65.79	-59.39	-56.59	-58.81
106	543.19	93.22	18.10	31.39	16.07	18.57	18.12	16.03	17.77
107	135.0A	58.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
108	830.54	180.36	48.99	140.62	50.07	48.97	55.74	48.69	43.24
111	1007.99	136.95	123.99	123.99	122.12	130.10	123.83	124.00	124.40
112	820.66	125.65	-73.11	-73.11	-74.65	-78.04	-73.14	-72.67	-69.28
113	918.62	129.06	-123.30	-123.30	-121.52	-124.13	-122.56	-121.74	-124.60
114	930.57	132.33	-61.67	-50.42	-59.38	-54.99	-62.00	-63.48	-62.54
116	591.60	105.40	-11.44	*** 110.30	-11.75	-11.36	-11.83	-12.55	-9.05
117	864.07	125.54	-0.32	-54.35	-2.09	0.35	-1.02	-1.74	-1.94
118	770.27	128.78	-34.33	6.37	-34.81	-36.24	-33.84	-33.17	-32.55
119	810.98	136.84	2.56	-89.65	-0.31	3.60	3.02	0.07	3.61
120	970.29	146.55	73.58	*** 152.36	76.60	74.90	74.18	74.19	77.42
121	527.31	96.56	*** -138.01	-26.90	*** -143.88	*** -136.62	*** -137.72	*** -137.52	*** -138.86
122	727.80	121.15	48.77	*** 198.35	47.89	40.22	48.07	48.65	48.85
123	1218.64	173.64	140.07	143.65	109.25	144.42	139.95	141.89	138.63
124	826.76	143.76	-34.65	-34.65	-33.92	-37.89	-34.92	-38.21	-34.60
125	809.88	142.29	-81.01	-81.01	-58.58	-81.41	-80.92	-74.67	-77.35
126	805.97	141.94	*** -154.44	*** -154.44	*** -153.51	*** -155.27	*** -154.53	*** -156.31	*** -156.11
127	1024.99	160.07	118.90	118.90	116.55	117.33	118.73	123.42	113.00
128	995.08	157.72	-132.76	-132.76	-123.41	-130.87	-132.14	-133.47	-131.68
129	707.88	133.03	-83.81	-83.81	-76.68	-72.88	-83.91	-84.98	-83.29
130	773.79	133.45	-40.03	-55.45	-56.75	-50.41	-39.84	-38.27	-38.75
131	1149.40	164.92	14.84	30.26	25.81	7.49	14.92	-15.33	16.18
132	897.45	149.78	34.73	34.73	28.39	59.86	35.00	40.58	34.75
133	740.08	133.28	16.47	-5.08	3.89	2.95	18.39	32.29	19.23
134	941.39	153.41	-11.69	-11.69	-1.76	14.17	-11.88	-18.22	-12.33
135	1009.70	158.87	76.10	76.10	100.27	73.51	75.81	85.86	74.52
137	842.40	138.87	110.43	103.20	119.56	106.13	110.13	112.65	117.85
138	738.25	124.11	-80.26	-120.08	-82.07	-82.45	-80.44	-90.64	-82.83
139	932.73	146.87	*** 149.43	*** 173.04	*** 152.17	142.38	*** 149.88	*** 170.37	*** 151.98
140	1134.48	168.40	74.43	74.43	136.63	77.90	74.57	73.32	78.02
141	1117.73	167.16	-130.35	-130.35	*** -175.12	-128.94	-130.58	-125.24	-130.63
146	1133.80	168.36	*** 254.06	*** 254.06	*** 238.79	*** 249.72	*** 254.48	*** 249.42	*** 258.32
147	1481.08	191.12	*** -210.87	*** -210.87	-184.50	*** -208.42	*** -211.52	*** -214.90	*** -213.98
151	2103.20	229.30	-4.92	-4.92	15.21	-4.31	-4.61	-14.46	-4.53
152	1533.20	195.78	-85.85	-85.85	-96.06	-91.49	-86.30	-85.62	-67.69
153	1329.82	178.42	70.80	103.82	66.05	68.94	71.05	67.42	67.10

Table 4 (con't)

LOOP CLOSURES (MM.)

LOOP	DIST. (KM.)	ALLOW (MM.)	ALL CORR.	W/O MAG.	W/O REF.	W/O ROD.	W/O LEVEL.	W/O TEMP.	W/O ASTRO.
154	1553.13	180.71	-67.28	47.25	-64.81	-65.29	-66.82	-68.16	-68.87
156	2101.19	212.67	22.70	122.19	20.01	21.89	22.86	44.40	24.04
160	950.78	137.34	-90.19	-90.19	-81.94	-94.31	-92.65	-90.74	-84.66
161	772.10	137.48	133.10	133.10	115.92	*** 146.60	132.44	132.98	136.71
162	574.45	119.83	-29.32	-29.32	-30.33	-32.20	-29.14	-23.09	-31.85
163	909.00	142.32	*** -179.72	-68.03	*** -176.31	*** -187.77	*** -179.32	*** -181.91	*** -176.84
164	1034.24	150.35	*** 215.04	*** 236.29	*** 202.90	*** 215.84	*** 217.11	*** 216.59	*** 223.92
165	813.88	130.43	*** -208.14	*** -208.14	*** -197.67	*** -200.93	*** -206.85	*** -208.20	*** -215.88
166	770.55	134.87	48.67	48.67	58.38	47.21	49.24	47.29	47.30
167	991.63	152.26	75.74	75.74	57.82	72.03	75.51	73.82	80.23
168	1129.57	150.01	-8.67	*** -190.07	-2.93	-8.71	-5.52	-6.73	-10.85
169	1147.04	154.70	*** -290.14	*** -355.92	*** -281.68	*** -288.06	*** -291.91	*** -291.51	*** -294.92
170	1351.70	167.61	136.90	136.90	120.74	144.50	135.04	139.38	145.71
171	1465.31	165.56	5.89	5.89	14.33	-2.91	3.13	2.46	7.97
173	1085.76	147.84	*** 149.70	*** 149.70	141.45	*** 150.58	*** 149.47	149.43	147.20
175	1792.75	188.52	*** -421.35	*** -421.35	*** -422.63	*** -417.07	*** -423.55	*** -424.31	*** -421.07
177	1367.35	176.16	92.42	92.42	90.79	86.58	92.02	101.29	86.41
182	817.63	136.10	101.41	101.41	106.22	104.21	102.22	99.74	102.26
183	797.98	129.80	55.12	55.12	53.89	55.58	54.80	54.60	58.64
184	418.04	92.99	-5.29	-5.29	-4.20	-6.60	-4.98	-6.31	-4.78
188	1569.26	195.83	*** 258.77	*** 258.77	*** 276.28	*** 235.55	*** 256.11	*** 254.26	*** 260.58
189	1247.02	143.75	60.77	130.29	54.98	56.24	62.41	59.85	63.99
190	1167.61	164.93	104.80	104.80	116.46	110.85	103.85	109.88	102.07
191	285.43	50.68	-1.86	11.21	-0.90	-1.66	-1.83	-1.18	-2.24
192	1104.01	155.69	*** -178.88	*** -178.88	*** -168.59	*** -186.75	*** -176.70	*** -170.90	*** -186.52
193	1355.00	216.19	-101.94	-101.94	-188.99	-28.51	-110.05	-96.01	-99.83
194	1661.29	203.79	*** -260.57	*** -260.57	*** -249.41	*** -302.07	*** -259.91	*** -267.98	*** -262.06
195	2226.40	219.60	-103.27	-103.27	-128.46	-97.34	-102.01	-117.03	-100.07
196	1636.96	202.29	113.44	113.44	116.83	94.16	113.23	107.30	111.79
198	1267.91	178.03	-88.64	-88.64	-73.58	-84.97	-89.11	-92.78	-88.73
200	3331.90	289.20	131.63	131.63	157.17	142.41	130.93	139.76	150.41
209	1606.06	200.37	-43.77	-43.77	-36.94	-52.60	-44.53	-36.48	-49.14
210	1830.44	213.91	80.11	80.11	63.19	95.42	80.24	87.08	84.59
211	1851.81	215.16	115.09	115.09	75.86	109.05	115.59	139.56	112.43
212	1000.48	155.80	-35.28	-11.72	-9.90	-34.22	-35.04	-52.05	-39.64
213	793.38	140.83	104.19	104.19	104.03	49.86	104.31	99.66	105.88
214	1199.19	173.14	*** -181.81	*** -181.81	-144.16	-160.97	*** -182.40	*** -175.49	*** -183.88
215	1354.76	184.03	-160.43	-160.43	-139.07	-163.85	-161.24	-115.27	-158.21
218	2336.13	233.97	142.66	142.66	124.65	117.82	143.94	161.52	150.44
221	1840.42	214.50	-207.76	-207.76	-211.38	-207.64	-206.98	-248.31	-208.10
222	1546.40	196.62	82.91	82.91	83.40	103.69	83.42	68.97	81.21
223	2060.37	222.56	179.19	179.19	190.56	185.66	178.24	174.09	178.91
224	1013.89	159.20	-56.95	-56.95	-40.71	-67.98	-56.95	-37.48	-59.39
225	2143.66	231.49	101.49	101.49	101.07	88.37	100.95	112.32	97.08
226	1318.11	176.86	16.61	16.81	18.78	13.30	17.98	19.77	18.96
227	1979.26	213.36	*** -447.10	*** -447.10	*** -423.45	*** -423.26	*** -448.01	*** -435.99	*** -446.61
228	1280.27	178.90	144.81	144.81	94.23	171.72	144.72	119.87	146.00
229	820.42	138.15	-11.04	-11.04	-21.00	-3.10	-10.54	-14.85	-10.13
230	627.38	102.51	0.04	42.43	-7.25	35.04	-0.39	-7.11	0.00
231	749.50	112.55	30.23	-44.57	42.26	3.93	29.41	29.57	29.52
232	619.18	106.04	-63.33	*** -119.87	-45.71	-83.41	-62.97	-65.35	-65.36
234	482.24	98.09	26.94	64.46	24.04	42.55	27.03	35.67	29.15
235	567.98	100.30	*** -121.45	*** -230.89	*** -122.55	*** -147.10	*** -121.25	*** -104.76	*** -118.76
236	611.91	98.94	75.18	*** 195.05	86.52	75.82	75.87	73.02	74.81

10d

Table 4 (con't)

LOOP CLOSURES (MM.)

LOOP	DIST. (KM.)	ALLOW (MM.)		ALL CORR.		W/O MAG.		W/O REF.		W/O ROD.		W/O LEVEL.		W/O TEMP.		W/O ASTRO.
237	772.03	111.14		31.03	***	-172.13		29.30		20.89		30.88		34.28		35.21
238	1220.92	172.95	***	-178.77		-164.18	***	-169.07		-169.03	***	-180.45	***	-192.51	***	-178.93
239	701.57	132.43		-82.45		-82.45		-38.80		-72.53		-82.48		-83.75		-83.72
241	883.05	130.87		34.65		34.65		13.44		24.30		34.45		37.74		38.32
242	1211.15	166.98		109.89		109.89		94.52		134.07		111.15		89.01		108.10
243	827.15	141.63		-18.72		-29.64		-21.75		-13.74		-16.91		-15.31		-17.70
244	1171.56	167.88	***	225.69	***	225.89	***	271.02	***	173.52	***	225.63	***	248.76	***	224.95
245	2087.92	222.18	***	265.61	***	265.61	***	238.69	***	243.48	***	266.65	***	276.58	***	287.22
246	1479.59	192.32		92.35		92.35		82.44		99.91		92.62		96.30		89.55
250	1180.04	183.71		-157.91		-157.91		-174.08		-127.37		-161.25		-159.47		-154.00
251	828.88	142.90		-53.35		-53.35		-28.18		-54.03		-52.99		-43.90		-53.24
252	892.32	163.56		69.21		69.21		47.14		50.27		67.37		34.22		69.72
254	1338.73	173.87		48.98		48.98		132.33		-16.07		48.87		17.34		50.45
255	888.49	146.28		-93.53		-93.53	***	-151.09		-14.81		-91.96		-92.69		-93.40
256	881.87	128.63	***	-178.64	***	-178.64	***	-184.18	***	-185.60	***	-178.17	***	-176.65	***	-176.63
257	845.80	175.78		-95.02		-109.68		-76.91		-101.90		-94.59		-100.59		-94.88
258	481.85	147.28		-88.88		-120.50		-84.43		-58.74		-86.96		-61.03		-65.12
52A	28.10	24.03		6.70		-4.92		6.88		6.48		6.86		6.74		6.91
114A	35.81	29.92		-5.19		-5.18		-5.26		-5.55		-5.18		-5.20		-5.18
114B	198.58	70.46		-19.03		-19.03		-19.65		-19.04		-19.28		-19.09		-20.13
115A	499.25	90.37		-46.51		-77.81		-46.31		-46.78		-46.19		-46.53		-45.98
115B	510.28	123.45		-5.99		-46.39		-4.78		-13.32		-6.12		-4.20		-5.29
115C	479.19	124.43		-17.37		-17.37		-18.50		-14.99		-16.64		-17.87		-14.99
115X	107.29	48.31		5.63		-7.20		5.41		6.86		5.91		5.59		6.60
136A	405.51	80.54		34.03		-0.89		31.55		23.76		34.02		31.53		33.22
188A	376.28	89.22		40.75		70.33		39.77		41.63		38.48		40.38		39.44
188B	308.77	79.97		48.15	***	84.75		48.64		47.70		48.37		47.79		45.47
189A	602.60	98.19		-2.20		45.75		-0.24		-2.25		-0.18		-1.91		-1.72
184A	266.15	72.90		-16.48		-16.48		-15.93		-16.54		-15.98		-15.60		-14.70
232A	384.19	83.79		9.83		-36.29		7.35		31.75		9.81		7.28		8.41
235A	284.75	69.00		46.55	***	111.43		50.34	***	83.72		46.38		41.72		46.06
236B	384.36	78.42		-37.38		-31.65		-46.29		-37.70		-37.19		-37.91		-37.68
236C	320.82	71.62		-7.20		68.37		-7.53		-21.45		-6.96		-17.15		-7.16
236D	773.92	111.27	***	-136.37		-18.01	***	-133.08	***	-125.55	***	-136.61	***	-152.93	***	-136.68
243A	322.19	71.79	***	91.21	***	103.61	***	92.41	***	90.58	***	90.77	***	91.58	***	91.61
244A	176.96	97.77		42.08		42.08		59.39		43.51		42.24		39.25		41.56
252A	350.28	114.90	***	-168.03	***	-168.03	***	-128.35	***	-166.20	***	-166.88	***	-134.89	***	-168.40
257A	170.54	89.43	***	102.69	***	102.69	***	95.56	***	102.83	***	103.08	***	102.81	***	101.45

Table 5

10e

LOOP CLOSURE DIFFERENCES (MM.)
CORRECTION 1 MINUS -----

LOOP	DIST (KM.)	ALLOW. (MM.)	CORR. 2	CORR. 3	CORR. 4	CORR. 5	CORR. 6	CORR. 7
1	763.22	132.36	-145.21	-1.31	0.78	-1.19	0.06	-2.68
2	658.32	124.76	-38.06	-0.50	-0.26	0.50	-1.44	0.71
3	860.06	142.76	-81.26	14.40	-4.88	1.19	1.57	2.58
4	1305.18	166.48	---	-11.04	3.22	0.22	6.48	5.87
6	1517.21	182.97	---	5.02	8.77	-2.35	-2.13	0.36
7	1002.49	158.31	---	-0.53	-9.19	-0.24	2.74	-2.39
8	891.99	144.18	-1.51	-1.48	6.17	0.58	-3.03	1.48
9	456.46	102.83	-14.36	-2.09	-5.47	-1.53	-0.48	0.77
10	568.73	113.98	1.17	-11.54	-7.50	1.19	-0.89	2.12
11	760.36	122.02	-60.53	-1.55	0.29	-1.53	-1.33	-3.54
12	531.11	99.13	45.13	4.16	-0.10	-0.12	0.16	0.44
13	448.99	96.55	-1.80	2.62	7.95	0.11	0.81	-1.21
14	523.72	114.42	42.34	2.01	-0.27	0.32	0.12	0.03
15	389.83	94.63	-61.65	1.05	-1.72	-0.71	3.84	3.61
16	527.39	111.53	40.66	2.91	4.11	-0.46	-4.36	0.64
17	876.48	144.53	-19.55	2.14	-0.57	0.06	0.89	-1.85
18	968.00	140.77	---	-4.75	-9.25	0.70	3.05	4.92
19	1084.20	153.83	---	14.22	-6.51	1.08	-2.52	-6.49
1A	282.30	80.97	67.60	-0.03	0.10	0.24	0.03	0.50
1B	592.89	137.94	167.71	-0.05	0.05	0.54	0.08	0.55
20	647.88	125.22	---	-5.79	2.24	-0.25	-6.25	-2.24
21	573.19	104.60	---	-11.59	1.11	-0.53	0.22	2.17
22	729.70	123.31	21.21	12.80	-14.87	-1.54	0.17	1.01
23	850.88	137.03	25.16	-25.42	12.13	-0.10	-6.62	-1.82
24	678.96	129.72	-9.98	16.23	-1.86	-0.11	1.52	-0.65
25	755.74	114.41	-82.33	-4.98	-3.27	-1.01	-4.17	6.31
26	860.38	123.10	-19.16	-2.19	1.31	0.51	2.65	-2.59
27	669.57	124.80	-98.75	5.70	4.32	0.93	0.49	-0.78
28	594.70	112.91	89.37	-2.53	-0.78	1.77	2.03	0.93
29	568.22	110.53	---	-1.53	0.12	0.86	-3.01	3.57
30	242.42	77.85	---	0.16	-0.35	0.09	0.91	0.10
31	700.92	129.42	---	-3.42	-3.22	0.71	3.06	-1.48
32	540.07	110.89	38.70	-5.71	0.61	-0.29	-0.66	2.12
36	526.83	105.90	10.05	-2.55	0.24	-0.81	0.29	0.30
37	178.27	61.50	---	-0.20	0.25	-0.10	1.50	0.00
38	641.78	121.41	-26.72	-5.05	7.11	-1.53	-1.83	0.37
50	632.15	119.08	160.31	0.24	2.97	1.03	0.86	2.04
51	555.97	117.89	-108.53	-2.37	-2.39	-0.43	-0.65	0.12
52	791.91	140.70	-51.80	0.21	-1.50	-0.18	-5.75	-1.73
53	802.77	136.36	-111.47	7.48	1.21	-0.23	8.10	1.86
54	1071.42	152.37	-196.27	-4.18	5.26	0.95	3.17	5.51
55	703.19	118.52	---	-3.64	2.07	-0.05	1.42	2.40
56	313.74	70.85	24.90	0.99	0.26	1.23	0.03	0.03
57	641.46	110.42	47.32	0.48	0.53	-2.66	0.53	0.81
58	312.83	78.20	-71.75	-0.11	-1.10	0.58	0.76	-0.08
59	502.52	102.69	---	-0.11	1.21	-1.29	-0.78	0.14
60	683.11	115.41	---	1.95	2.39	-1.18	0.50	1.99
61	832.18	132.51	-33.99	-2.91	-2.08	-0.45	-0.43	-5.29
62	697.40	114.06	33.99	-1.86	-0.33	0.23	-0.04	4.49
63	594.13	97.49	-148.53	-1.13	-0.87	-0.85	1.51	-0.65
70	705.06	128.57	---	-3.25	8.93	-0.60	1.77	1.48
71	589.35	110.09	---	1.11	6.06	2.42	0.35	2.44
72	962.88	142.26	---	5.19	-0.50	-1.62	1.24	-3.87
73	1638.86	193.83	---	1.40	2.97	0.25	-0.34	-0.76

COR. 1 - All Correction Applied
COR. 2 - All Corrections except magnetic
COR. 3 - All Corrections except refraction
COR. 4 - All Corrections except rod scale
COR. 5 - All Corrections except collimation
COR. 6 - All Corrections except rod temp.
COR. 7 - All Corrections except astronomic

--- No magnetic error in loop

Table 5 (con't)

LOOP CLOSURE DIFFERENCES (MM.)
CORRECTION 1 MINUS -----

LOOP	DIST (KM.)	ALLOW. (MM.)	COR. 2	COR. 3	COR. 4	COR. 5	COR. 6	COR. 7
77	1574.75	160.62	106.91	-11.71	-2.16	-1.10	0.31	2.69
79	911.55	133.90	-43.80	6.43	-1.72	0.95	-1.03	-0.19
80	685.04	130.86	---	-4.94	7.34	-0.12	-3.02	-2.40
81	875.94	140.29	91.63	4.07	3.59	-0.98	-1.93	2.13
82	902.65	150.22	---	-7.18	1.46	0.25	1.80	-0.69
84	765.16	138.30	---	6.65	-0.29	0.01	-0.87	-0.18
86	719.71	134.13	---	-10.14	1.11	0.25	-0.05	1.14
87	720.47	134.20	---	-9.52	-0.13	-0.39	4.41	2.59
88	918.54	151.37	---	-27.13	-2.30	0.25	-11.78	6.92
90	1390.93	186.47	1.03	19.73	20.68	-0.50	5.97	0.23
93	884.66	148.71	-1.02	-6.96	-8.34	0.08	-3.14	4.10
94	1074.22	163.87	-0.14	5.69	-16.66	-0.33	7.17	2.39
97	918.17	151.50	---	-26.25	2.37	-0.60	-12.81	0.61
98	1041.83	161.38	---	-37.93	-11.15	0.48	2.76	-3.49
101	749.44	130.38	---	28.26	19.62	-0.54	-6.43	-1.47
102	1076.86	156.81	---	-27.95	-21.34	0.13	-5.69	3.32
103	817.75	134.58	82.24	-2.10	7.08	0.68	-2.12	0.10
106	543.19	93.22	-13.29	2.03	-0.47	-0.02	2.07	0.33
107	135.09	58.11	---	0.00	0.00	0.00	0.00	0.00
108	830.54	180.36	-91.63	-1.08	0.02	-8.75	0.30	5.75
111	1007.99	136.95	---	1.87	-6.11	0.16	-0.01	-0.41
112	820.66	125.65	---	1.74	4.93	0.03	-0.44	-3.85
113	918.82	129.06	---	-1.78	0.83	-0.74	-1.56	1.30
114	930.57	132.33	-11.25	-2.29	-6.68	0.33	1.79	0.87
116	591.60	105.40	-121.74	0.31	-0.08	0.39	1.11	-2.39
117	884.07	125.54	54.03	1.77	-0.87	0.70	1.42	1.62
118	770.27	128.78	-40.70	0.48	1.91	-0.49	-1.16	-1.78
119	810.98	136.84	92.21	2.87	-1.04	-0.46	2.49	-1.05
120	970.29	146.55	-78.78	-3.02	-1.32	-0.60	-0.61	-3.84
121	527.31	96.56	-111.11	5.87	-1.39	-0.29	-0.49	0.85
122	727.60	121.15	-151.58	-0.92	6.55	0.70	-1.88	-2.08
123	1218.64	173.64	-3.56	30.82	-4.35	0.12	-1.82	1.44
124	828.76	143.78	---	-0.73	3.24	0.27	3.56	-0.05
125	809.88	142.29	---	-22.43	0.40	-0.09	-6.34	-3.66
126	805.97	141.94	---	-0.93	0.83	0.09	1.87	1.67
127	1024.99	160.07	---	2.35	1.57	0.17	-4.52	5.90
128	995.06	157.72	---	-9.35	-1.89	-0.62	0.71	-1.08
129	707.88	133.03	---	-7.13	-10.93	0.10	1.17	-0.52
130	773.79	133.45	15.42	18.72	10.38	-0.19	-1.76	-1.28
131	1149.40	164.92	-15.42	-10.97	7.35	-0.08	30.17	-1.34
132	897.45	149.78	---	6.34	-25.13	-0.27	-5.85	-0.02
133	740.08	133.28	23.55	14.58	15.52	0.08	-13.82	-0.76
134	941.39	153.41	---	-9.93	-25.86	0.19	6.53	0.64
135	1009.70	158.87	---	-24.17	2.59	0.29	-9.76	1.58
137	842.40	138.87	7.23	-9.13	4.30	0.30	-2.22	-7.42
138	738.25	124.11	39.80	1.79	2.17	0.16	10.36	2.35
139	932.73	146.87	-23.61	-2.74	7.05	-0.45	-20.94	-2.55
140	1134.46	168.40	---	-62.20	-3.47	-0.14	1.11	-3.59
141	1117.73	167.16	---	44.77	-3.41	0.23	-5.11	0.28
146	1133.80	168.38	---	15.27	4.34	-0.42	4.64	-2.26
147	1461.08	191.12	---	-28.37	-2.45	0.65	4.03	3.11
151	2103.20	229.30	---	-20.13	-0.61	-0.31	9.56	-0.39
152	1533.20	195.78	---	10.21	5.64	0.45	-0.23	2.04
153	1329.62	178.42	-33.02	4.75	1.86	-0.25	3.38	3.70

Table 5 (con't)

LOOP CLOSURE DIFFERENCES (MM.)
CORRECTION 1 MINUS -----

LOOP	DIST (KM.)	ALLOW. (MM.)	COR. 2	COR. 3	COR. 4	COR. 5	COR. 6	COR. 7
154	1553.13	180.71	-114.53	-2.47	-1.99	-0.46	0.88	1.59
156	2101.19	212.67	-99.49	2.69	0.81	-0.16	-21.70	-1.34
160	950.78	137.34	---	-8.25	4.12	2.46	0.55	-5.53
161	772.10	137.48	---	17.18	-13.50	0.66	0.12	-3.61
162	574.45	119.63	---	1.01	2.88	-0.18	-6.23	2.53
163	909.00	142.32	-111.69	-3.41	8.05	-0.40	2.19	-3.08
164	1034.24	150.35	-21.25	12.14	-0.80	-2.07	-1.55	-8.88
165	813.88	130.43	---	-10.47	-7.21	-1.29	0.06	7.74
166	770.55	134.87	---	-9.71	1.46	-0.57	1.36	1.37
167	991.63	152.26	---	17.92	3.71	0.23	2.12	-4.49
168	1129.57	150.01	181.40	-5.74	0.04	-3.15	-1.94	2.18
169	1147.04	154.70	65.78	-8.46	-2.08	1.77	1.37	4.78
170	1351.70	167.61	---	16.16	-7.60	1.86	-2.48	-8.81
171	1465.31	165.56	---	-8.44	8.60	2.76	3.43	-2.08
173	1065.76	147.84	---	8.25	-0.88	0.23	0.27	2.50
175	1792.75	188.52	---	1.28	-4.28	2.20	2.96	-0.28
177	1367.35	176.16	---	1.63	5.84	0.40	-8.87	6.01
182	817.63	136.10	---	-4.81	-2.80	-0.81	1.67	-0.85
183	797.98	129.80	---	1.23	-0.46	0.52	0.52	-3.52
184	418.04	92.99	---	-1.09	1.31	-0.31	1.02	-0.51
186	1569.26	195.83	---	-17.51	23.22	2.66	4.51	-1.79
189	1247.02	143.75	-69.52	5.79	4.53	-1.64	0.92	-3.22
190	1167.61	164.93	---	-11.66	-6.05	0.95	-5.08	2.73
191	285.43	50.66	-13.07	-0.96	-0.20	-0.03	-0.68	0.38
192	1104.01	155.89	---	-10.29	9.87	-2.18	-7.98	7.64
193	1355.00	216.19	---	87.05	-73.43	8.11	-5.93	-2.11
194	1661.29	203.79	---	-11.16	41.50	-0.66	7.41	1.49
195	2226.40	219.80	---	25.10	-5.93	-1.26	13.76	-3.20
196	1636.96	202.29	---	-3.30	19.28	0.21	6.14	1.65
198	1267.91	178.03	---	-15.06	-3.67	0.47	4.14	0.09
200	3331.90	289.20	---	-25.54	-10.78	0.70	-8.13	-18.78
209	1606.06	200.37	---	-6.83	8.83	0.76	-7.29	5.37
210	1830.44	213.91	---	16.92	-15.31	-0.13	-6.97	-4.48
211	1851.81	215.16	---	39.23	6.04	-0.50	-24.47	2.66
212	1000.46	155.80	-23.56	-25.38	-1.06	-0.24	16.77	4.36
213	793.38	140.83	---	0.16	54.33	-0.12	4.53	-1.69
214	1199.19	173.14	---	-37.65	-20.84	0.59	-6.32	2.07
215	1354.76	184.03	---	-21.36	3.42	0.81	-45.16	-2.22
218	2336.13	233.97	---	18.01	24.84	-1.28	-18.86	-7.78
221	1840.42	214.50	---	3.62	-0.12	-0.78	40.55	-1.66
222	1546.40	196.62	---	-0.49	-20.78	-0.51	13.94	1.70
223	2060.37	222.56	---	-11.37	-6.49	0.95	5.10	0.28
224	1013.89	159.20	---	-16.24	11.03	0.00	-19.47	2.44
225	2143.66	231.49	---	0.42	13.12	0.54	-10.83	4.41
226	1316.11	176.86	---	-1.97	3.51	-1.17	-2.96	-2.15
227	1979.26	213.36	---	-23.65	-23.84	0.91	-11.11	-0.49
228	1280.27	178.90	---	50.58	-28.91	0.09	24.94	-1.19
229	820.42	136.15	---	9.96	-7.94	-0.50	3.81	-0.91
230	627.38	102.51	-42.39	7.29	-35.00	0.43	7.15	0.04
231	749.50	112.55	74.80	-12.03	26.30	0.82	0.66	0.71
232	619.18	106.04	56.54	-17.62	20.08	-0.36	2.02	2.03
234	482.24	98.09	-37.52	2.90	-15.61	-0.09	-8.73	-2.21
235	567.98	100.30	109.44	1.10	25.65	-0.20	-16.69	-2.69
236	611.91	98.94	-119.67	-11.34	-0.64	-0.49	2.16	0.37

Table 5 (con't)

Kgal-mm

LOOP CLOSURE DIFFERENCES (MM.)
CORRECTION 1 MINUS -----

LOOP	DIST (KM.)	ALLOW. (MM.)	COR. 2	COR. 3	COR. 4	COR. 5	COR. 6	COR. 7
237	772.03	111.14	203.16	1.73	10.34	0.15	-3.25	-4.18
238	1220.92	172.95	-14.59	10.30	-9.74	1.68	13.74	-1.84
239	701.57	132.43	---	-23.65	10.08	0.03	1.30	1.27
241	683.05	130.67	---	21.21	10.35	0.20	-3.09	-3.67
242	1211.15	166.98	---	15.37	-24.18	-1.26	20.88	3.79
243	827.15	141.63	12.92	5.03	-2.98	0.19	-1.41	0.98
244	1171.56	167.88	---	-45.33	52.17	0.06	-21.07	0.74
245	2067.92	222.16	---	26.92	22.13	-1.04	-10.97	-1.61
246	1479.59	192.32	---	9.91	-7.56	-0.27	-3.95	2.80
250	1160.04	183.71	---	16.17	-30.54	3.34	1.56	-3.91
251	626.66	142.90	---	-27.19	0.68	-0.36	-9.45	-0.11
252	892.32	163.56	---	22.07	18.94	1.84	34.99	-0.51
254	1338.73	173.87	---	-83.35	65.05	0.11	31.84	-1.47
255	866.49	146.28	---	57.56	-78.72	-1.57	-0.84	-0.13
256	661.87	128.63	---	7.54	8.96	1.53	0.01	-0.01
257	845.80	175.78	14.66	-18.11	6.88	-0.43	5.57	-0.14
258	481.65	147.28	53.62	-2.45	-8.14	0.08	-5.85	-1.76
52A	28.10	24.03	11.62	-0.18	0.22	-0.16	-0.04	-0.21
114A	35.81	29.92	-0.01	0.07	0.36	-0.03	0.01	-0.01
114B	198.58	70.46	---	0.62	0.01	0.25	0.06	1.10
116A	499.25	90.37	31.30	-0.20	0.27	-0.32	0.02	-0.53
115B	510.26	123.45	40.40	-1.21	7.33	0.13	-1.79	-0.70
115C	479.19	124.43	---	1.13	-2.38	-0.73	0.50	-2.38
115X	107.29	46.31	12.63	0.22	-1.03	-0.28	0.04	-0.97
138A	405.51	80.54	34.92	2.48	10.27	0.01	2.50	0.81
166A	376.28	89.22	-29.58	0.98	-0.88	2.27	0.37	1.31
168B	308.77	79.97	-36.60	-0.49	0.45	1.78	0.36	2.68
169A	602.60	96.19	-47.95	-1.96	0.05	-2.02	-0.29	-0.48
184A	268.15	72.90	---	-0.55	0.06	-0.50	-0.68	-1.78
232A	364.19	83.79	46.12	2.48	-21.92	0.02	2.55	1.42
235A	284.75	69.00	-64.88	-3.79	-37.17	0.17	4.83	0.49
236B	384.36	78.42	-5.73	8.91	0.32	-0.19	0.53	0.30
236C	320.62	71.62	-75.57	0.33	14.25	-0.24	9.95	-0.04
236D	773.92	111.27	-120.36	-3.29	-10.82	0.24	16.56	0.31
243A	322.19	71.79	-12.40	-1.20	0.63	0.44	-0.37	-0.40
244A	176.96	97.77	---	-17.31	-1.43	-0.16	2.83	0.52
252A	350.28	114.90	---	-39.68	20.17	0.85	-31.34	0.37
257A	170.54	89.43	---	7.13	0.06	-0.39	-0.12	1.24

were applied. Second, of the 28 loop misclosures that were outside their allowable limits when all corrections were applied, only 4 were within the allowable limits when the magnetic correction was not applied, i.e., loop numbers 121, 163, 236D, and 238. Next, of the same 28 loops that were outside their allowable limits when all corrections were applied, only 2 were within allowable limits when the refraction correction was not applied, i.e., loop numbers 147 and 214. Therefore, in general, the corrections did not adversely affect the loop misclosures.

Table 5 lists the differences between loop misclosures computed when all corrections were applied for systematic errors and loop misclosures computed when all corrections except those for a particular correction were applied. For example, column 4 in table 5 is the difference in loop misclosures between when all corrections were applied minus when all corrections were applied except the magnetic correction. Some other interesting items are noted in table 5. First, 96 loops were influenced by magnetic error. Of these, 23 were outside their allowable limits when the magnetic correction was not applied; only 9 loops were outside their allowable limits when the magnetic correction was applied. Therefore, the modeled magnetic correction appears to be working reasonably well. Second, the accumulated refraction correction in the loops was usually less than 2 cm. (See column 5, table 5.) Some loops located in mountainous regions accumulated slightly more refraction correction, i.e., loops 140 (-6.2 cm), 228 (5.1 cm), 254 (-8.3 cm), and 255 (5.8 cm). However, these differences are still relatively small for the size of the loops; e.g., loop 254 is 1,338.73 km. This was expected because the refraction correction usually accumulates when height differences are large between points. Third, the accumulated influence of the rod correction was usually small; i.e, its total accumulation was usually less than 1 cm. (See column 6, table 5.) Once again, there were a few loops in the mountains which accumulated larger differences, i.e, loop numbers 213 (5.4 cm), 244 (5.2 cm), 254 (6.5 cm), and 255 (-7.9 cm). Lastly, the remaining corrections--level collimation (column 7, table 5), temperature (column 8, table 5), and astronomic correction (column 9, table 5)--had an insignificant influence on the loop misclosures, i.e, their accumulative effect was always less than a few centimeters.

Results of Adjustment Analyses

After all loop misclosures were analyzed, bench mark heights were computed using a minimum-constraint least squares adjustment. A total of 36 links were rejected because of large residuals and large loop misclosures. Most of these links involved connections between "old" and "new" leveling data. Table 2 lists some general statistics from the minimum-constraint least squares adjustment. Figures 6-14 give more specific details from the results of the adjustment.

All corrections applied

The first adjustment performed was a minimum-constraint least squares adjustment holding fixed the height of a tidal bench mark, referenced to a zero value of the 1960-78 local mean sea level, at Key West,FL. This station was selected as the constraint to prevent negative heights for other LMSL, but any station could have been used. The height was referenced to the 1960-78 LMSL so all other adjusted heights of tidal bench marks could be compared with their corresponding heights above LMSL.

Figure 6 gives the differences between heights computed from the minimum-constraint least squares adjustment and published NGVD 29 heights at the junction bench marks. Referring to figure 6, an east-to-west systematic difference between the adjusted heights and the published NGVD 29 heights seems to exist. This accumulates to a significant difference of about 160 cm from Maine to Washington. In addition, the difference reaches about 200 cm at some junction bench marks located in the Rocky Mountains.

Figure 7 shows the differences between heights estimated from the minimum-constraint least squares adjustment and heights above the 1960-78 LMSL at primary tidal stations. Also appearing in figure 7 are the east-to-west systematic differences; e.g., from Portland, ME, to Seattle, WA, the difference is 158 cm. Differences along the U.S. Atlantic coast do not indicate a systematic tilt between geodetic leveling and local mean sea level. The difference between the two surfaces at Fernandina Beach, FL, and Portland, ME, was only 4.5 cm. There were, however, a few large tilts over relatively short distances. For example, the difference from Fernandina Beach to Charleston, SC, was 11 cm over a distance of 450 km; and from Charleston, SC, to Duck, NC, the difference was -11 cm over a distance of 550 km.

There is, however, a large apparent tilt of 50 cm along the west coast extending from San Diego, CA, to Neah Bay, WA. NGS recently releveled most of the west coast. This latest releveling may shed some light on why the differences on the west coast appear to be systematic, while the differences on the east coast do not.

The positive difference between geodetic leveling and tidal data from the east to west coasts is about twice the stated oceanographic value. Oceanographers indicate that MSL of the eastern Pacific Ocean is about 70 cm higher than MSL of the western Atlantic Ocean (Montgomery 1969). Therefore, they would expect the leveled difference from tidal bench marks on the east coast to tidal bench marks on the west coast to disagree with heights above LMSL by approximately 70 cm. The recently determined geodetic differences vary, depending on which east-west station pairs are considered, but they are all positive and greater than 70 cm. Using station pair Fernandina Beach, FL, and San Diego, CA, the difference is 108 cm; using station pair Duck, NC, and Crescent City, CA, which are both fairly open-ocean stations, the difference is 136 cm; and using station pair Portland, ME, and Seattle, WA, the difference is 158 cm.

Considering that the leveling route distances from the east coast to the west coast range between 5,000 km and 7,500 km, the first two differences stated above seem reasonable. Over such long distances, remaining systematic errors in the leveling data could account for 50 to 75 cm. In addition, the estimated standard error of the 70-centimeter estimate for the height difference between the mean sea levels of the two oceans is at least 10 cm (Sturges and Montgomery, 1974). The fact that the differences are all positive tends to indicate that systematic errors remain in the leveling data. Also, the effects of local sea surface topography upon tidal stations (Merry and Vanicek 1983, Vanicek et al. 1985), such as the tidal station at Seattle, WA, located in Puget Sound, may account for differences of as much as 20 cm. Considering the large extent of the vertical control network and the thousands of individual setups required to level across the country, differences of 50 to 75 cm seem reasonable.

Adjustments with and without Corrections Applied for Systematic Errors

It is difficult to separate the overall change in bench mark heights into individual components such as the effects of systematic errors, crustal movements, and datum distortions. In order to estimate the influence of systematic errors on adjusted heights, adjustments were performed both with and without corrections applied for systematic errors. Figures 8-13 depict differences in adjusted heights estimated with and without certain corrections applied to the data. The plots indicate the amount of influence the correction has on the adjusted height at the junction bench marks. Comparisons of adjusted heights, with and without corrections applied, indicate that, except for the magnetic correction, they do not significantly change the adjusted heights in a continental sense. However, in some regions they do have a large local effect.

Figure 8 clearly indicates that magnetic correction has a significant effect on the adjusted heights of bench marks. It reaches about 50 cm at the Canadian border. Note that the effect in the east-west direction is small. This was expected because the error due to the Earth's magnetic field in some automatic compensator-type leveling instruments, e.g., some older models of the Zeiss Ni 1, reaches significant proportions when leveling in a north-south direction.

Comparison of least squares adjusted heights, with and without magnetic correction applied to the data, demonstrates an unfavorable aspect of least squares adjustments, i.e., the smoothing effect. Before an adjustment is performed on leveling data, all systematic errors and blunders should be removed from the data. In general, the lower the degrees of freedom, i.e., low redundancy, the more difficult it is to detect data outliers. The remaining errors--random, systematic, and blunders--are distributed throughout the data by the adjustment process. Leveling networks usually consist of low degrees of freedom and remaining systematic errors are distributed depending on the loop misclosures. Loops that contain systematic errors will usually have larger misclosures than loops that do not contain systematic errors. However, a least squares adjustment will distribute a portion of the error to every observation, regardless of whether the data were or were not actually contaminated by systematic error.

To show that a least squares adjustment can distribute error in the wrong places, differences in observed height differences were compared with differences in adjusted height differences, with and without magnetic correction applied. Figure 8 depicts the differences in adjusted heights with and without the magnetic correction applied. Figure 8a shows a detailed section from Camden, NC, to St. Augustine, FL. It is interesting to note that the estimate of accumulated magnetic correction between bench marks V 176 and X 223 is -77 cm, while the estimate of magnetic error from adjusted heights is only -37 cm. This means the adjustment distributed the remaining 40 cm of error into "good" data. This is one reason why influences of systematic effects should be removed from data prior to performing leveling adjustments. Additional parameters can be added to the observation equations to account for systematic effects, as long as an appropriate model, describing the error source, can be developed and sufficient redundancy exists to solve for these parameters. If this procedure is properly performed, the adjustment should not distribute the systematic effects into "good" data.

Figure 9 depicts the influence of refraction correction on adjusted heights. It is obvious that the correction does not significantly change adjusted heights in a continental sense; i.e., from coast to coast the influence is almost zero. There are, however, some local differences between heights corrected and not corrected for refraction which are significantly large; e.g., in the mountains of California and Oregon, a few relative differences exceed 10 cm. (See figure 9.) Once again, this difference was expected because most junction bench marks are located in valleys, not on mountain summits where the refraction correction can accumulate to larger amounts.

Such leveling lines usually begin at a junction bench mark in a valley, go up one side of the mountain and down the other side to another junction bench mark in a valley. The local accumulation of the refraction correction at the top of the mountain is usually larger than the overall accumulation. These local effects will be investigated separately.

Figures 10-12 indicate that the effects of the rod, level, and temperature corrections, respectively, on adjusted heights are generally insignificant locally, as well as globally. Figure 13, however, which depicts the influence of the astronomic correction, indicates that the influence of this correction on adjusted heights is larger than many of the others. Figure 13 shows that the astronomic correction is not significant locally, but globally it is larger than the others, except for the magnetic correction. Figures 8-13 also indicate that the total of all the corrections is not large enough to account for the large coast-to-coast systematic tilt between 1988 adjusted heights from this study and published NGVD 29 heights.

The next step in the study was to investigate the influence due to "true" geopotential differences using real gravity, instead of normal orthometric height differences based on normal gravity. In the NGVD 29 adjustment, normal orthometric corrections, which were based on normal gravity, were applied to the leveling data. Figure 14 depicts differences between adjusted orthometric heights computed using geopotential differences (based on observed gravity with orthometric heights estimated using Helmert's reduction formula) and adjusted normal orthometric heights (based on normal gravity). Once again, the differences are not significant in a continental sense, i.e., from the east coast to the west coast the overall difference is only 5 to 6 cm. Locally, however, the effect in the mountains reaches about 50 cm. These differences, however, do not explain the systematic differences between published NGVD 29 heights and minimum-constraint least squares adjusted heights from the special primary vertical control network. In an adjustment of leveling data, errors are distributed throughout the network depending on loop misclosures. The NGVD 29 readjustment was no exception. The next step was to investigate the NGVD 29 readjustment project results.

ANALYSES OF NGVD 29

It would have been very helpful to the datum definition study to recreate the 1929 general adjustment using geopotential differences. However, this was not possible, because a majority of the original data used in the NGVD 29 adjustment was not placed in computer-readable form. Many of the original leveling lines were releveled and because the old leveling was not essential to the readjustment project, these older data were not automated. Figure 15 depicts leveling data used in the 1929 adjustment which were placed in computer-readable form. It is obvious from reviewing figure 15 that a network cannot be recreated, not even a single leveling line, from coast to coast using the 1929 data in the NGS data base.

However, in support of NAVD 88, NGS' Vertical Network Branch converted the historic height difference links involved in the 1929 General Adjustment to computer-readable form. The 1929 General Adjustment was recreated by constraining the heights of the original 26 coastal stations. Free adjustment results were then compared with the General Adjusted constrained results. Several differences exceeded 50 cm. A very large relative difference, 86 cm, was detected between St. Augustine, Florida, and Fort Stevens, Oregon. This is indicative of the amount of distribution and present in the 1929 General Adjustment (See figures 15a and 15b).

Because a complete network consisting of geopotential differences from the 1929 adjustment could not be generated, leveling data used in the 1929 adjustment along a single-line leveling route from Seattle, WA, to Crookston, MN, were combined and an adjustment performed. The refraction correction was not applied to the data because it was not applied in 1929.

Figure 16 shows the vertical control used in the 1929 general adjustment, including the leveling route from Seattle, WA, to Crookston, MN. A minimum-constraint least squares adjustment was performed on this single line to obtain a set of heights for the bench marks. The 1929 general adjustment height value of a bench mark in Seattle was held fixed in the new adjustment. Figure 17 plots the differences between the adjusted heights estimated from the single-line leveling route and the heights obtained from the 1929 general adjustment. There appears to be a systematic difference in these two sets of heights, which accumulates to more than 100 cm. This large difference is surprising because these data were used to compute the heights in the 1929 general adjustment. In an ideal situation, if the loop misclosures for these lines were zero and appropriate constraints were applied, then these two sets of heights should be equal and figure 17 should not show a tilt. The only explanation for this large tilt is that very large corrections were distributed over these leveling lines in 1929. In fact, after further investigation, this is what actually happened. Figure 18 gives the 1929 link-by-link distribution corrections from Seattle to Crookston. The total distribution correction was 89 cm over a distance of 3,000 km. This agrees fairly well with figure 17 and accounts for part of the systematic differences between adjusted heights of the new primary vertical control network and published NGVD 29 heights. (See figure 6.) These large distribution corrections could be due to the distribution of large loop misclosures and/or constraints used in the 1929 adjustment.

Next, the original computations of the 1929 general adjustment were obtained and analyzed. These included observed differences in height, distribution corrections, loop misclosures, list of rejected leveling links, and height constraints used in the 1929 adjustment. Maps depicting link numbers, loop numbers, observed differences in heights, and 1929 distribution corrections (residuals) were generated. Figures 19-20 depict the distribution of loop misclosures normalized by their allowable tolerances. This ratio takes into account the length of the loop. FGCC specifications of first-order, class II loop tolerances were used to compute the allowable limits. Table 6 lists some general statistics of the loop misclosures. From figures 18-20 and table 6, it is obvious that large adjustment corrections were applied to the northwest U.S. data in the 1929 General Adjustment. The errors implied by large misclosures were at least in part distributed throughout the network.

Table 6.--Summary of statistics based on NGVD 29 loop misclosure analysis

	No. of loops by sign			No. of loops outside allowable limit			No. of loops within allowable limit		
	Neg.	Pos.	Total	Neg.	Pos.	Total	Neg.	Pos.	Total
All data included	142	139	281	28	30	58	114	109	223
Outliers removed	137	134	271	20	22	42	117	112	229

An interesting fact, and one that no doubt pleased analysts in 1929, is that the minimum-constraint least squares adjusted heights of the 1929 vertical control network agreed fairly well from coast to coast with the 1929 tidal differences. The differences between heights at tidal bench marks estimated from the 1929 special adjustment and heights estimated from local mean sea level data differed by only 36 cm from Portland, ME, to Seattle, WA. This close agreement between local mean sea level values and geodetic leveling from coast to coast implies that constraining the heights of 26 tidal stations to their local mean sea level values did not cause these large adjustment corrections. However, that same type of comparison using adjusted heights estimated from the 1988 primary vertical control network and the 1960-78 local mean sea level tidal epoch was 158 cm. The difference of 122 cm (158 cm minus 36 cm) is too large to be accepted as "noise" in the leveling data, especially when such large loop misclosures were present in the 1929 adjustment. This led to the next phase of the analysis which was the comparison of height differences using the 1929 single-line leveling routes with height differences using the 1988 single-line leveling routes.

SINGLE-LINE LEVELING ROUTES COAST-TO-COAST ANALYSIS: 1929 VERSUS 1988

To investigate the coast-to-coast leveling differences between 1929 and 1988 data, five single-line leveling routes were selected. They were (1) Portland, ME, to Seattle, WA; (2) Atlantic City, NJ, to Seattle, WA, (3) Atlantic City, NJ, to San Francisco, CA, (4) Norfolk, VA, to San Francisco, CA, and (5) Fernandina, FL, to San Pedro, CA. Figure 21 shows these leveling routes. These were not five independent routes. Some routes used some of the same data, as shown in figure 21. The 1988 leveling routes were generated using data from the special primary vertical control network discussed in this report. Wherever possible, the 1988 routes were designed to match the 1929 routes. The 1929 leveling data were compiled from observations listed in the general adjustment report. The differences were then added to estimate the differences between tidal stations. Table 7 lists the differences between tidal stations using the two sets of data.

Differences were estimated in two ways: (1) orthometric height difference minus appropriate local mean sea level tidal height difference at tidal bench marks and (2) orthometric height difference between common tidal bench marks. Several interesting results are noted in table 7. First, the differences between the NAVD 88 and NGVD 29 single-line leveling routes from coast to coast agree better than the adjusted height differences estimated from the network adjustments. That is, the differences in table 7 labeled "dH_{88} - dH_{29}" are less than 100 cm, whereas the adjusted height differences estimated from the network adjustments are greater than 120 cm. In fact, the 1988 geodetic height differences minus the tidal height differences agree to better than 86 cm. Considering that the leveling route distances range from 5,000 to 7,500 km, these differences seem reasonable, although the differences are all positive, which implies systematic errors remaining in the 1929 and/or 1988 data. This will be addressed in more detail later in the report.

The next result to note in table 7 is that all 1929 leveling height differences minus the 1929 tidal height differences are greater than the 36 cm estimated from the 1929 special minimum-constraint least squares adjustment. As a matter of fact, they range from 56 to 147 cm, depending on the leveling route. This implies that the leveling network adjustment distributed loop misclosures throughout the network in such a manner that it reduced the single-line leveling minus tidal height difference of 81 cm, from Portland to Seattle, to 36 cm. This explains some of the large distribution corrections in the northwestern United States. The last result to note in table 7 is that 1929 geodetic height differences minus 1929 tidal differences are all closer to the 70 cm height difference that oceanographers say is a reasonable estimate of height difference between the two oceans. This is surprising, considering the fact that the instruments and rods used prior to 1929 were less accurate than 1988 equipment and that 1929 procedures were not as strict as the procedures used in 1988, e.g., sight lengths were limited to 150 m in 1929, while they were limited to 60 m in the newer data. Still, obtaining agreements of 32 cm and 86 cm between the new and old data over distances up to 7,500 km lends credibility to the leveling data, at least to the 50-to-75 cm level of uncertainty.

Table 7.--Leveling height differences (dH) minus local mean sea level differences (dT): 1988 and 1929 (values in parentheses are without refraction correction applied to data)

Sta. to Sta.	$dH_{88} - dT_{88}$ (m)	$dH_{29} - dT_{29}$ (m)	$dH_{88} - dT_{29}$ (m)	$dH_{29} - dT_{29}$ (m)
	$dH_{88-29} - dT_{88-29}$ (m)		$dH_{88} - dH_{29}$ (m)	
Portland to Seattle	1.5632 (1.6222)	0.8083	1.5543 (1.6226)	0.8083
(difference)	0.7549 (0.8139)		0.7460 (0.8143)	
Atlantic City to Seattle	1.7800 (1.8464)	1.4652	1.8723 (1.9387)	1.4652
(difference)	0.3148 (0.3812)		0.4071 (0.4735)	
Atlantic City to S.F.	1.5657 (1.5599)	0.7095	1.6787 (1.6630)	0.7095
(difference)	0.8562 (0.8504)		0.9692 (0.9535)	
Norfolk to S.F.	1.4103 (1.4284)	0.5644	1.5089 (1.5270)	0.5644
(difference)	0.8459 (0.8640)		0.9445 (0.9626)	
Fernandina to San Pedro	1.0201 (1.0318)	0.5891	1.1316 (1.1433)	0.5891
(difference)	0.4310 (0.4427)		0.5425 (0.5542)	

In addition to the coast-to-coast leveling differences between 1929 and 1988 data, three north-south single-line leveling routes were investigated. Figure 21a depicts these leveling routes. Once again, the 1988 leveling routes were generated using data from the special primary vertical control network and the 1929 leveling data were compiled from observations listed in the general adjustment report. Corrections to account for refraction and astronomic effects were removed from the 1988 leveling data to be comparable with the 1929 leveling data.

The comparison of the 1988 and 1929 three north-south, single-line leveling routes are given on figure 21a. The differences clearly indicate that the old and new leveling data agree within a 35-cm level of uncertainty.

SINGLE-LINE LEVELING ROUTES FROM SEATTLE WA, TO CROOKSTON, MN, ANALYSIS: 1988 VERSUS 1929

To investigate the differences between the 1929 and 1988 data, leveling data from Seattle, WA, to Crookston, MN, were compared. Data for a single-line leveling route were generated for each epoch, and bench mark heights were determined from least squares adjustments. The height of a tidal bench mark in Seattle, common to both epochs, was held fixed in the minimum-constraint least squares adjustment. Figures 22-23 depict the leveling routes selected. The two routes are the same except from Pasco, WA, to Butte, MN. To evaluate the differences in adjusted heights, plots of differences in adjusted heights were generated.

Figures 24-25 are plots of the terrain for the 1988 and 1929 routes, respectively. Note that from west longitudes 119 to about 114 degrees, the heights are different because, as stated above, the routes are different in this region. Figure 26 depicts the differences in adjusted heights between the two epochs. All data were corrected for the influence of known systematic effects. Adjusted heights were obtained using geopotential differences based on gravity values derived from the 4-kilometer gridded Bouguer anomaly data set of the Society of Exploration Geophysicists. This plot is interesting because it shows a systematic difference between the two plots, which appears to be terrain dependent. However, even with the apparent systematic difference, the overall difference between the starting and ending bench marks is less than 25 cm for a leveling distance of 3,000 km. Even though there is an obvious systematic difference, this difference seems to be reasonable considering the distance.

Figure 27 is a plot of height differences using the normal orthometric correction, instead of geopotential differences. The overall difference does not change significantly. Figure 28 shows the adjusted height differences without corrections for systematic effects. Note that the differences exceed 100 cm.
Figure 29 plots the differences in adjusted heights using data with all systematic corrections applied except refraction correction. Note that the overall difference is reduced by a factor of two, but the apparent systematic tilt is still present.

Since a portion of 1988 data contained magnetic error, a plot of the differences in adjusted heights with all corrections applied except magnetic error was generated. (See figure 30.) Once again, the overall difference did not change very much. This difference was expected because the leveling route was mainly in a west-east direction.

Figures 31, 32, and 35-42 depict differences in adjusted heights for each epoch with and without certain corrections applied to the data. The refraction correction (figures 31-32) accumulated positively when leveling up the mountains and negatively when going back down the mountains. However, the total accumulation of the refraction correction in the 1929 data is more than 10 cm (figure 32), while the accumulation in the 1988 data is about -5 cm (figure 31). The 1929 leveling data were observed along railroad tracks where refraction effects can be large. In addition, the average sight length of the 1929 data set was 85.6 m, while the average sight length for the 1988 data set was only 36.6 m. (See tables 8 and 9.)

Table 8.--Average sight lengths of leveling lines used in 1929 single-line leveling route from Seattle, WA, to Crookton, MN

Leveling line accession number	Approximate height difference (m)	Length of line (km)	Average sight length (m)
70698	3	198	85.4
57499	102	402	53.2
70893	1,567	898	77.5
68775	-1,010	815	81.1
70892	-87	258	75.3
68633/5	-161	36	101.0
68633/4	-120	132	113.4
68633/3	-17	92	113.4
68633/2	8	136	81.5
68633/1	-202	152	74.0
		Average	85.6

The refraction correction equation currently used by NGS assumes leveling is performed along the shoulder of a highway. The temperature gradient along a railroad is usually larger than along a shoulder. Holdahl (1982) estimated that the correction along a railroad may be underestimated by 42.5 percent. Therefore, two additional plots were generated: one with the refraction correction increased by 42.5 percent and, to be conservative, one with the correction increased by 25 percent. Figures 33-34 depict the differences between 1988 data and modified 1929 data. Modifying the refraction correction for the 1929 data decreased the tilt going down the mountain, but did not decrease the overall difference of 25 cm. As a matter of fact, it increased the difference slightly.

Table 9.--Average sight lengths of leveling lines used in 1988 single-line leveling route from Seattle, WA, to Crookston, MN

Leveling line accession number	Approximate height difference (m)	Length of line (km)	Average sight length (m)
L23133	21	38	35.4
L23136	267	41	37.4
L24471/2	338	101	29.7
L24472/1	40	210	36.5
L21444	-604	66	33.6
L21441	0	26	30.9
L21526	204	260	33.1
L24469/3	13	25	35.3
L24475/2	665	335	42.4
L24475/1	9	19	39.3
L24479/4	688	198	31.7
L24479/7	-462	107	34.1
L24479/6	-179	157	32.1
L24481/2	-25	139	34.3
L24481/3	-312	320	40.0
L24484/3	-58	384	41.2
L24485/2	-17	65	41.8
L24486/4	-139	220	35.0
L24487/3	-2	119	42.0
L24489/3	-29	69	39.7
L24489/2	-176	247	42.2
		Average	36.6

The interesting plots in figures 35-36 show the effect of applying rod corrections. Please note that different vertical scales were used to plot the results. The scales differ by a factor of 10. Accumulated rod correction in the 1929 data approaches 80 cm, while the 1988 data accumulates to only 5 cm. The leveling rods used for the 1929 data were "paraffin-soaked" wood and had extremely large rod excess values. (Rod excess is an average value of the differences between actual and nominal lengths of the leveling rod.) The wooden rods were always calibrated in the laboratory at the beginning and end of each leveling season. In addition, they were checked in the field during the season. The rod excess values appear to change from year to year (Strange 1982). It is easy to understand how an error in an estimated rod excess could cause the differences between the 1988 and 1929 leveling data. The differences appear to be terrain dependent, as is true for the rod excess correction.

Figures 37-42 give the effects on differences in adjusted heights when the remaining corrections are applied to the data. Figures 37-38 show the effects due to applying the level correction. Figures 39-40 show the effects due to applying the temperature correction, and figures 41- 42 show the effects due to applying the astronomic correction. They are all small corrections and the plots confirm this. Figures 41-42 are interesting because the corrections are opposite in sign, but they still differ by only 5 cm.

Analysis of leveling data along the single-line route from Seattle to Crookston indicated that the 1988 and 1929 levelings agree with each other to within 25 cm over a distance of 3,000 km. This agrees fairly well with the single-line coast-to-coast routes which show differences between 32 and 86 cm. These differences imply that, at the present time, this may be the best estimate of a leveling height difference from coast to coast. The apparent systematic differences between old and new leveling data over these long distances could easily be due to unmodeled and/or remaining systematic effects in the leveling data. This also indicates that the large systematic height difference of 150 cm from coast to coast between the primary vertical control network and the published NGVD 29 heights is due mostly to a large distribution of corrections in the 1929 adjustment results.

DIFFERENT DATUM DEFINITION SCENARIOS

To assist in the datum definition decision, several adjustments were performed using different constraints. In addition to the minimum-constraint least squares adjustment discussed previously, four more adjustments, using different constraints, were performed: (1) the heights of bench marks above LMSL 1960-78 at Key West, FL, and Portland, ME, were held fixed; (2) the heights of bench marks above LMSL 1960-78 at Key West, FL, Portland, ME, Neah Bay, WA, and San Diego, CA, were held fixed; (3) the height of a bench mark above LMSL 1960-78 at Key West, FL, was held fixed and an observation of 70 gal-cm (standard error equal to 0.1 gal-cm) between the Duck, NC, tidal station, and the Crescent City, CA, tidal station was added to the data; and (4), same as (3) except the standard error of the observation between Duck and Crescent City was changed to 10 gal-cm.

Figures. 43, 45, 47, and 49 give the differences in adjusted heights between adjustment results for the NAVD 88 primary vertical control network using the four different constraints and published NGVD 29 heights. Looking at these figures, it is obvious that no matter which datum definition scenario is chosen, including minimum constraint, differences in heights between NGVD 29 and NAVD 88 of 75 to 100 cm will exist.

Figures 44, 46, 48, and 50 depict the differences in heights between adjustment results for the NAVD 88 primary vertical control network under the different constraints and the heights of tidal bench marks above LMSL 1960-78. Even constraining the heights of two tidal bench marks on each coast produced large (25 cm) differences between the adjusted heights and heights above LMSL 1960-78 for the special primary vertical control network.

Interestingly, figure 49 shows that an estimated difference of 70 gal-cm between the Atlantic and Pacific Oceans with its appropriate standard error of 10 gal-cm does not change the heights from coast to coast by more than 10 cm. Figure 51 shows the differences between the minimum-constraint least squares adjusted heights and the adjusted heights estimated with the 70 gal-cm height difference added to the data set (standard error equal to 10 gal-cm). The 70 gal-cm height difference with a standard error of 10 gal-cm did not significantly change the adjusted heights because geodetic leveling height differences from coast to coast indicate the difference should range between 110 and 160 cm, not 70 cm. The standard error of a leveling height difference from coast to coast is less than 10 cm.

SELECTION OF A DATUM

The obvious theoretical selection of the NAVD 88 datum is a variation of adjustment number 4 discussed in the previous section. This requires holding the height of one tidal bench mark referenced to the LMSL of the 1960-78 tidal epoch fixed (or minimizing the differences between specific tidal height values and NAVD 88 heights) and adding estimated height difference values between appropriate tidal stations with their corresponding standard errors.

The following questions need to be answered: Which tidal bench mark height should be held fixed or which stations should be involved in minimizing the differences between LMSL 1960-78 heights and NAVD 88? What is the "best" estimate of a height difference between a <u>tidal station</u> pair from the east coast to the west coast of the United States? Is the difference 70 cm, 60 cm, 80 cm, or 100 cm, or what? What is the "best" estimate of the standard error of the height difference between a tidal station pair coast to coast? Is the height difference 10 gal-cm, or 20 gal-cm, or what?

As previously mentioned, figures 49-50 indicate that adding the 70 gal-cm height difference with its appropriate standard error of 10 gal-cm does not significantly change the adjusted heights of the junction bench marks. (See figure 51.) It is important to note that this is based on the assumption that the relative weighting scheme of the data is correct. The standard error of leveling data over relatively "short" distances, i.e., 500 km, is probably appropriate. For first-order, class II, double-simultaneous, single-run leveling, the standard error of an observation is equal to 0.14 cm times the square root of the distance in kilometers;

therefore, the standard error of an observation over a distance of 500 km is 3.1 cm. When combining the same types of data in relatively small network adjustments, the relative weighting scheme is usually not considered to be a problem. In larger vertical control network adjustments, where all types of data, i.e., geodetic leveling, steric leveling, and tidal data, are combined, having a correct relative weighting scheme becomes extremely important.

The previous discussion leads to another set of questions. What are the systematic effects remaining in the leveling data? How large are the influences of these systematic effects? What are the influences of these systematic effects on the relative weighting scheme? If leveling height differences are influenced by remaining systematic effects, then the formal estimates of standard error for leveling height differences are too optimistic. If the standard errors of the leveling data are too optimistic, then the leveling data will have more of a controlling effect in the adjustment than they should, compared with the tidal height differences. The standard error of the leveling data could be increased and/or the standard error of the tidal height differences could be decreased. However, it is improper to arbitrarily change a priori standard errors of observations. A priori standard errors of observations should be estimated based on the estimates of errors that influence the method of obtaining the observations.

From a pragmatic point of view, if a correct relative weighting scheme cannot be determined, then appropriate weights should be selected which control the network in such a manner that the results conform to the "truest" scenario. For example, if it is believed that the leveling data still contain systematic errors that are larger than the uncertainties of the estimates of height differences between tidal station pairs, then the height differences between tidal station pairs should be given more weight to help control the remaining errors in leveling data. Of course, it must be understood that one assumption is being substituted for another.

DISCUSSION OF DATUM DEFINITION SELECTION

This report shows several plots which indicate that large differences between NAVD 88 and NGVD 29 heights will exist no matter how the 1988 datum is defined. As stated, these differences are due to many factors, such as large distribution corrections (residuals) from the NGVD 29 adjustment, better estimates of corrections applied to account for systematic errors, estimating geopotential differences using real gravity values instead of using normal orthometric height differences, and physical changes in monument heights between old and new leveling surveys. It should be noted that the NAVD 88 heights are better estimates of orthometric heights than are the NGVD 29 heights. This will become more critical in the future as surveying techniques continue to become more sophisticated and more accurate. The development of the Rapid Precision Leveling System (Lataitis et al. 1985) is an example of a future design that will be more accurate than present equipment. The improvement of geoid height determinations using published orthometric heights and GPS-derived ellipsoid heights requires the best estimate of "true" orthometric heights. The development of improved geoid models should use observed leveling data with gravity information and GPS data, not published orthometric height. The typical user, however, will be using published orthometric heights and GPS-derived ellipsoid height information. Many map makers also want heights on their maps based on the best estimate of "true" orthometric heights.

An important aspect which needs to be emphasized is that the changes in bench mark height that will result from the NAVD 88 readjustment are primarily due to better estimates of height differences, not height changes due to datum definition philosophy. No matter how the new datum is defined, some height values will change by 1 meter, or more; local relative differences in stable areas, however, will be small. The typical surveyor will not be significantly affected because the relative height changes between adjacent bench marks should be only a few millimeters. The absolute height values will change much more, but this should not be the surveyor's biggest concern. As discussed in the next paragraph, the surveyor's biggest problem will be ensuring that all height values of bench marks in the project are referenced to NAVD 88.

The 500,000 bench marks established by the USGS have not been placed in computer-readable form and will not have NAVD 88 heights. In addition, the U.S. Army Corps of Engineers (COE) has established hundreds of thousands of bench marks across the nation that will not have NAVD 88 heights. The Federal Geodetic Control Committee (FGCC) has established a Vertical Subcommittee to investigate the impact of NAVD 88 on the user community. The members of the subcommittee have been briefed on the results of this datum definition study and were requested to document their products and services that will be affected by the readjustment.

As implied above, NAVD 88 heights will be beneficial to users of the Global Positioning System (GPS) who are computing GPS-derived orthometric heights. A large error in estimating GPS-derived orthometric heights is the uncertainty in estimating geoid heights. There are many techniques and procedures which can be used to estimate relative GPS-derived orthometric heights with accuracies that are sufficient to meet many engineering needs (Vincenty 1987, Milbert and Holdahl 1988, Zilkoski and Hothem 1989). The new NAVD 88 adjustment will provide estimates of "true" orthometric height differences that will enable the average user to estimate relative GPS-derived orthometric heights to a sufficient accuracy to meet the requirements of many engineering projects.

Map makers such as USGS will probably be affected the most by the height differences between NGVD 29 and NAVD 88. As stated previously, USGS produces 60,000 different map products. The 7.5-minute series will be the one that is most affected by a datum change. There are approximately 100 spot elevations, which are given to the nearest foot (30 cm), on every 7.5-minute quad. The 7.5-minute quad series consists of various contour intervals: 14 percent contain 5-foot contours, 34 percent contain 10-foot contours, 29 percent contain 20-foot contours, 18 percent contain 40-foot contours, and 5 percent contain 80-foot contours (Southard 1985). According to Southard (1985), the national map standards for vertical accuracy require that 90 percent of the points tested will be accurate to within one-half the contour interval.

In addition, Southard also says that the survey closures on supplemental vertical control lines were required to be within one-tenth of the contour interval and spot elevations be within an accuracy of one-fourth the contour interval. Southard also says that a limited amount of change in the vertical datum can be allowed without recontouring a map. He suggested that one-tenth of the contour interval would be a reasonable amount.

In concluding, Southard states "A program for conversion of all maps of the NMP to the NAVD 88 would be prohibitively expensive. Adapting a new vertical datum to the NMP can however be accommodated by use of a datum change statement. This statement could be added at the time of reprinting, after the amount of the elevation change is known. A suitable statement could read 'To correct elevation on this map to the NAVD 88, add (subtract) _____ feet.'"

To assist USGS, as well as other users, NGS will compare published NGVD 29 heights with the new NAVD 88 heights to estimate a single bias factor which describes the difference between NGVD 29 and NAVD 88 for every 7.5-minute quad. These bias factors could be published in tables and distributed to all users. Computer programs using appropriately designed and validated data files could be developed to estimate a bias factor on a point-by-point basis. The accuracy of the bias shift will depend on the number of valid bench marks in the area of interest. The heights of some bench marks may have changed due to crustal movement or disturbance; these bench marks should not be used to estimate the bias factor.

There may be insufficient number of published bench marks in some 7.5-minute NAVD 88 quads to adequately estimate the bias shift. All bench marks presently published by NGS will have new NAVD 88 heights referenced to NAVD 88. However this does not include the third-order leveling performed by USGS and COE. If these data were placed into computer-readable form and incorporated into NAVD 88, then, in most areas of the country, a bias factor could probably be estimated with an uncertainty of 1 foot (30 cm) or better. If 86 percent of the 7.5-minute quads have 10-foot or larger contour intervals, then this bias factor should be sufficient. For the 14 percent which have contour intervals less than 10 feet, the bias factor and its uncertainty would have to be investigated carefully. In almost all cases where the 7.5-minute quad has a 5-foot or less contour interval, the terrain is relatively flat; areas of flat terrain usually have more leveling data than areas of steep terrain because the height component is more critical for evaluating flooding, construction, and drainage activities. Therefore, if all data are available, the bias factor in these areas may be estimated with a better uncertainty, perhaps to one-half foot (15 cm).

The psychological effects of the datum change along the ocean and gulf coasts may be harder to deal with than the bias shift. Figure 7 showed that differences between the minimum-constraint least squares adjustment of the primary network and tidal heights, referenced to 1960-78 LMSL, exceeds 1.6 m in the State of Washington. This means that a surveyor standing waist deep in water could still be 0.5 m above NAVD 88. This would also mean that the local mean sea level contour line would be 1.6 m above NAVD 88. Once again, a bias shift statement could be written on the map. But can the typical user accept the concept that values for the national geodetic vertical network differ from values for local mean sea level by more than 1.5 m? The alternative is to greatly distort the leveling data by fixing the heights of bench marks referenced to local mean sea level values.

It should be noted that during fiscal year 1990, NGS documented their position on the selection of NAVD 88 datum definition. A copy of the position paper is included in appendix of this report. According to USGS, "If the constrained elevation at Key West, Florida, is changed by -30 cm, then the map patching conversion can probably be applied to most (70-80 percent) of the 7.5-minute maps of the NMPs" (Chapman 1990). In order to minimize the effects on NMPs, as requested by users, NGS has selected the new International Great Lakes Datum local mean sea level height value at the primary water-level station at Father point/Rimouski as the minimum constraint for NAVD 88, i.e., the results shown in figure 3 in appendix A. See appendix A for more details.

COMPARISON OF PRIMARY NETWORK ADJUSTED HEIGHTS WITH HEIGHTS FROM OTHER SOURCES

Comparison of Primary Network Adjusted Heights with Preliminary Uncorrected Observed Canadian Heights

Figure 52 depicts the differences between adjusted heights estimated from this study and geodetic leveling height differences derived from uncorrected, observed Canadian data. The Geodetic Survey Division of Canada is still processing their leveling data, so these results are preliminary. Canadian heights from Maine to the west side of Lake Superior are based on a leveling network generated in support of an adjustment study of the International Great Lakes Datum of 1985 (IGLD 85). The IGLD 85 study is being performed jointly by the United States and Canada. Results of the analyses will be published in a separate report. Canadian heights from the west end of Lake Superior to Washington were estimated using uncorrected, single-line leveling differences. While local differences will change after corrections for systematic effects are applied to the data and additional observations added to form a network, it is anticipated that the overall differences should not increase by a large amount. If these differences do not change, then the overall difference between the U.S. network and the Canadian data will be about 61 cm. This is a reasonable difference, considering the leveling distance across the continent.

The Canadian data involved in the IGLD 85 study were influenced by magnetic effects. The Canadian Geodetic Survey Division performed a preliminary study documenting the effects of magnetic error on their leveling instruments (PVCS 1988). The study estimated the average magnetic constant to be -3.37 mm/km Gauss, which is similar to the average value of -3.68 mm/km Gauss determined by NGS (Holdahl et al. 1986). The average canadian magnetic constant was used to estimate the magnetic correction for the Canadian data. After applying the magnetic correction to the Canadian data of the IGLD network, the difference between the U.S. and Canadian IGLD networks was reduced from 13.8 cm to 0.8 cm and the coast-to-coast difference was reduced to 47.8 cm. (See figure 53.)

In addition, it was previously mentioned that the values west of Lake Superior were estimated using a single chain of border loops. When a second chain of Canadian loops was combined with the border loops to connect with the latest Trans-Canadian leveling line, the overall difference from Lake Superior to Washington decreased from 47 cm to 6 cm. The coast-to-coast difference between the U.S. and Canadian networks would then be less than 10 cm. (See figure 54.) One of the loops in the second chain had a large misclosure (-23 cm/1267 km); the closure is almost twice its allowable tolerance. The fact that the U.S. leveling network agrees with the Canadian network to less than 50 cm, with a possibility that this difference will decrease when all data are included to form a network and corrections are applied to account for known systematic effects, is encouraging. These differences will be updated and analyzed as additional data and corrections become available.

Comparison of Primary Network Adjusted Heights with Satellite-Derived Orthometric Heights

In a report by Despotakis (1987), brought to the attention of the authors by Prof. Richard H. Rapp, Ohio State University, numerical computations of geoid heights using several methods were compared with satellite-derived geoid heights (ellipsoid heights minus orthometric heights) at laser tracking stations distributed around the world.

The report states "The numerical computations of the geoid undulations using all the four methods resulted in agreement with the "ellipsoidal minus orthometric" value of the undulations on the order of 60 cm or better for most of the laser stations in the eastern United States, Australia, Japan, Bermuda, and Europe. A systematic discrepancy of about 2 meters for most of the western United States stations was detected and verified by using two relatively independent data sets. The cause of this discrepancy was not found."

The results of the datum definition study provides a possible explanation for this systematic discrepancy of 2 m in the western U.S. stations.

Table 10.-- Comparison of modeled geoid heights (N_m) with satellite-leveling- derived geoid heights (N_d) for western stations

Station name	N_m minus N_d		Difference in height
	OSU[1] (m)	MOD by NGS[2] (m)	special network minus NGVD 29[3] (m)
Platteville	-1.59	-0.11	1.481
Bear Lake	-1.99	-0.53	1.458
Fort Davis	-0.18	0.93	1.105
Otay Mt.	-2.29	-1.14	1.149
Mt. Laguna	-1.92	-0.62	1.302
Goldstone			
Mars	-2.11	-0.80	1.308
Venus	-2.38	-1.07	1.308
Owens Valley	-2.83	-1.14	1.695
Quincy	-2.03	-0.66	1.367
Mean	-1.92	-0.57	1.353
RMS	0.70	0.62	

[1] Values obtained using modified Sjoberg's method to estimate geoid undulation at the laser stations, including local average correction (Despotakis 1987: table 26, Page 96).
[2] Using OSU's value ([1]) plus NAVD minus NGVD 29 value ([3]).
[3] Based on results of this study at bench mark closest to the station.

As shown in figure 6, estimates of new heights located in the western United States were approximately 1.5 m higher than the heights published for NGVD 29. As far as the authors could determine from Despotakis' report and references, the orthometric heights for the U.S. stations in his report are referred to NGVD 29. If this is the case, then by substituting the estimates of orthometric heights from this datum definition study for the NGVD 29 values, the 2 m bias is reduced to 60 cm. This would be more consistent with the results of the stations around the world. Tables 10-11 list the comparisons between the modeled geoid heights and satellite-leveling-derived geoid heights using heights estimated from a minimum-constraint least squares adjustment of the primary vertical network (using the published NGVD 29 height value at the Key West tidal station which was held fixed.)

Table 11.-- Comparison of modeled geoid heights (N_m) with satellite-leveling-derived geoid heights (N_d) for eastern stations

Station name	N_m minus N_d OSU[1] (m)	N_m minus N_d MOD by NGS[2] (m)	Difference in height special network minus NGVD 29[3] (m)
Greenbelt 1	0.21	0.37	0.163
2	0.94	1.10	"
3	-0.23	-0.07	"
4	0.36	0.52	"
5	0.19	0.35	"
6	0.23	0.39	"
7	0.22	0.38	"
8	0.22	0.38	"
Patrick AFB	-0.73	-0.81	-0.077
Haystack	-1.00	-0.95	0.048
Mean	0.04	0.17	0.045
RMS	0.53	0.59	

[1] Values obtained using modified Sjoberg's method to estimate geoid undulation at the laser stations, including local average correction (Despotakis 1987: table 26, page 96).
[2] Using OSU's value ([1]) plus NAVD minus NGVD 29 value ([3]).
[3] Based on results of this study at bench mark closest to the station.

CONCLUSION

NGS is investigating the impact that NAVD 88 will have on the geodetic and mapping communities. Because each locale is unique, the impact will be slightly different in each region. A bias factor describing the approximate difference between NGVD 29 and NAVD 88 will be estimated by comparing bench mark heights in both systems. This factor should be sufficient for many users. NGS encourages all users to obtain a basic understanding of NAVD 88 and to express their concerns about the project. The more NGS understands users' needs, and the more users understand NAVD 88, the smoother the transition will be from NGVD 29 to NAVD 88.

To assist in identifying and documenting the impact of NAVD 88, NGS has undertaken this special study to compile a primary vertical geodetic network using the latest vertical control data available. Analyses of this network were helpful in determining the effects of various datum constraints, magnitudes of height changes from the NGVD 29 datum, influences of systematic errors, areas of crustal movement, deficiencies in network design, and additional releveling requirements.

It is difficult to separate overall change in bench mark heights into individual components, such as the effects of systematic errors, crustal movements, and datum distortions. Comparisons of adjusted heights, with and without corrections applied, indicate that, except for the magnetic correction, the adjusted heights are not significantly changed in a global sense, but in some regions they do have a large local effect.

The obvious selection of a theoretical NAVD 88 datum is an adjustment holding the height of one tidal bench mark referenced to the 1960-78 LMSL fixed (or minimizing the differences between specific tidal height values and NAVD 88 heights) and adding estimated height differences between LMSL at appropriate tidal stations with their appropriate standard errors.

The following questions should be answered. Which LMSL should be held fixed or which stations should be involved in minimizing the differences between heights above LMSL of 1960-78 and NAVD 88? What is the "best" estimate of a height difference between LMSL at tidal station pairs from the east coast to the west coast of the United States? What is the "best" estimate of the standard error of the height difference between LMSL's at a tidal station pair coast to coast?

When combining the same types of data in relatively small network adjustments, the relative weighting scheme is usually not considered to be a problem. In larger vertical control network adjustments where all types of data, i.e., geodetic leveling, steric leveling, and tidal data, are combined, having a correct relative weighting scheme becomes extremely important.

This leads to another set of questions to be answered. What are the systematic effects remaining in leveling data? How large are the influences of these systematic effects? What are the influences of these systematic effects on the relative weighting scheme? If leveling height differences are influenced by remaining systematic effects, then the formal estimates of standard error for leveling height differences are too optimistic. If the standard error of the leveling data are too optimistic, then the leveling data will have more of a controlling effect in the adjustment than they should, compared with other observations. The standard error of the leveling data could be increased and/or the standard error of the tidal and steric height differences could be decreased. However, it is improper to arbitrarily change a priori standard errors of observations. The estimates of a priori standard errors of observations should be based only on the estimates of errors that influence the method of obtaining the observations.

This report gives several plots showing the large differences between NAVD 88 and NGVD 29 heights that will exist regardless of which datum definition scenario is chosen for NAVD 88. These differences are due to many factors, such as large distribution corrections (residuals) from the NGVD 29 adjustment, better estimates of corrections applied to account for systematic errors, crustal movement, and estimating geopotential differences using real gravity values instead of normal orthometric height differences. The new 1988 heights are much better estimates of orthometric heights than are the NGVD 29 heights.

Users of orthometric heights require accurate heights referenced to the geoid. This will become more critical in the future as surveying techniques continue to become more sophisticated and accurate. The improvement of geoid height determinations using published orthometric heights and GPS-derived ellipsoid heights require the best estimate of "true" orthometric heights. It should be noted that the development of improved geoid models should use observed leveling data with gravity information and GPS data, not published orthometric heights. The typical user, however, will be using published orthometric heights and GPS-derived ellipsoid height information. Many map makers also want heights on their maps based on the best estimate of "true" orthometric heights.

Finally, an important aspect to emphasize is that changes in bench mark height values that will result from the new NAVD 88 adjustment are primarily due to better estimates of height differences, not a change in datum definition philosophy.

Once again, it should be noted that during fiscal year 1990, NGS documented their position on the selection of NAVD 88 datum definition. A copy of the position paper is included in appendix A of this report.

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